TWELVE BASIC FUNCTIONS

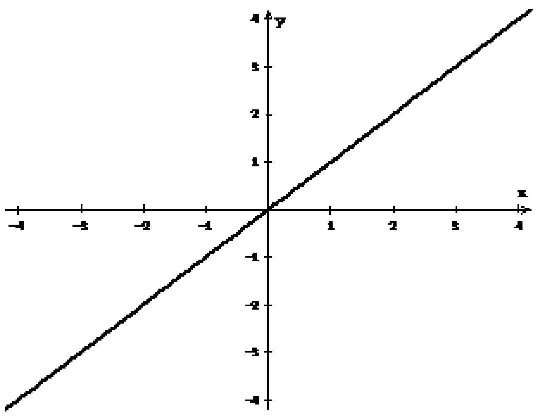
In advanced mathematics you will find it helpful to analyze functions that appear repeatedly. This lesson will help you recognize basic properties and characteristics of common functions.

DIRECTIONS

Give a complete analysis for each of the twelve basic functions. The analysis should include as many of the following as possible:

- **Domain**
- **Range**
- **Continuity**
- **Increasing/decreasing behavior**
- **Symmetry**
- **Boundedness**
- **Local extrema**
- **Horizontal asymptotes**
- **Vertical asymptotes**
- **End behavior**
- ***x*-intercepts**
- ***y*-intercepts**

The Identity Function



$$f(x) = x$$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Roots: $x = 0$

y-intercept: $(0, 0)$

Increasing intervals: $(-\infty, \infty)$

Decreasing intervals: none

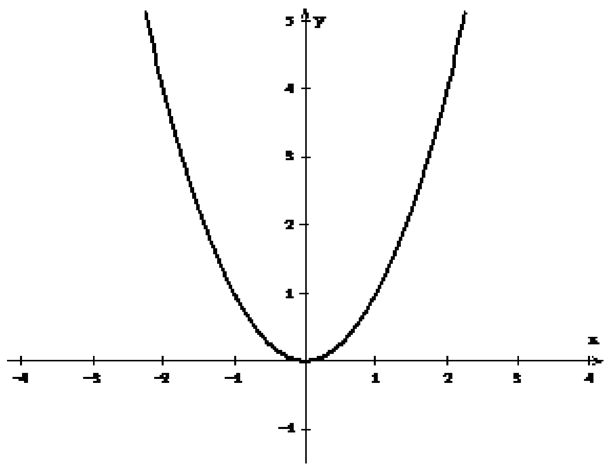
Relative max/min: none

Continuous?: yes Even/odd? : odd

Other: falls left rises right Bounded?: not

End behavior: $\lim_{x \to +\infty} f(x) = +\infty$ $\lim_{x \to -\infty} f(x) = -\infty$

The Squaring Function



$$f(x) = x^2$$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Roots: $x = 0$

y-intercept: $(0,0)$

Increasing intervals: $[0, \infty)$

Decreasing intervals: $[-\infty, 0]$

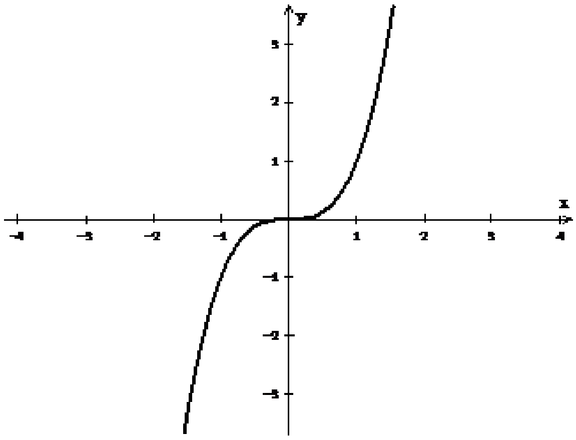
Relative max/min: absolute min @ $(0,0)$

Continuous?: yes Even/odd? : even

Other: rises on left and right Bounded?: below

End behavior: $\lim_{x \to +\infty} f(x) = +\infty$ $\lim_{x \to -\infty} f(x) = +\infty$

The Cubing Function



$$f(x) = x^3$$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Roots: $x = 0$

y-intercept: $(0,0)$

Increasing intervals: $(-\infty, \infty)$

Decreasing intervals: none

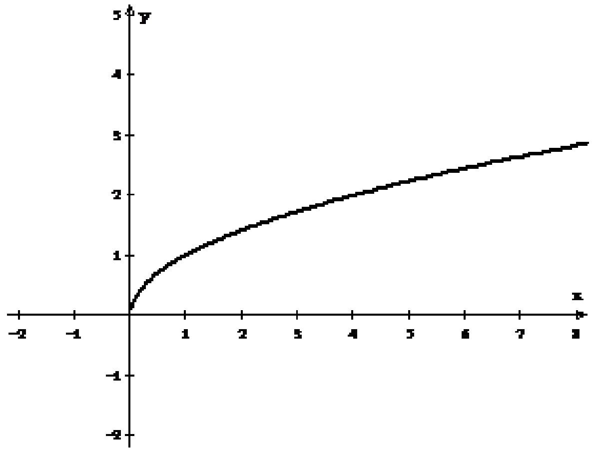
Relative max/min: none

Continuous?: yes Even/odd? : odd

Other: falls left, rises right Bounded?: no

End behavior: $\lim_{x\to+\infty} f(x) = +\infty$ $\lim_{x\to-\infty} f(x) = -\infty$

The Square Root Function



$f(x) = \sqrt{x}$

Domain: $[0, \infty)$

Range: $[0, \infty)$

Roots: $x = 0$

y-intercept: $(0,0)$

Increasing intervals: $[0, \infty)$

Decreasing intervals: none

Relative max/min: absolute minimum @ $(0,0)$

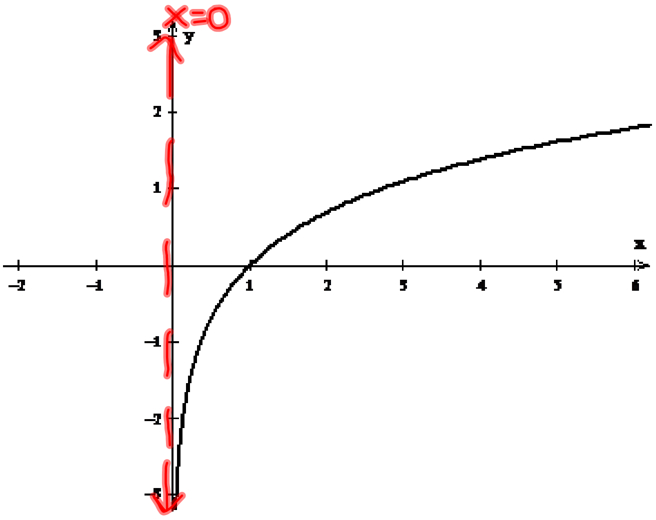
Continuous?: yes Even/odd? : neither

Other: ______ Bounded?: below

End behavior: $\lim_{x \to +\infty} f(x) = +\infty$ $\lim_{x \to -\infty} f(x) = \text{D.N.E.}$

D.N.E. means does not exist

The Natural Logarithm Function



$$f(x) = \ln x$$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Roots: $x = 1$

y-intercept: none

Increasing intervals: $(0, \infty)$

Decreasing intervals: none

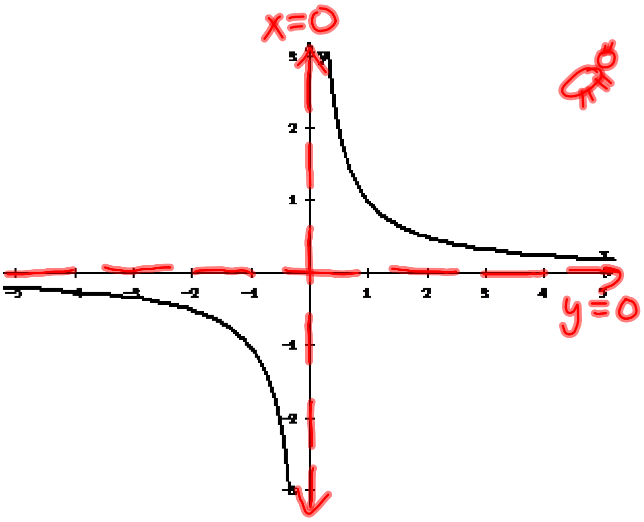
Relative max/min: none

Continuous?: yes Even/odd? : neither

Other: vertical asymptote $x=0$ Bounded?: no

End behavior: $\lim_{x \to +\infty} f(x) = +\infty$ $\lim_{x \to -\infty} f(x) =$ D.N.E.

The Reciprocal Function



$$f(x) = \frac{1}{x}$$

Domain: $(-\infty,0) \cup (0,\infty)$

Range: $(-\infty,0) \cup (0,\infty)$

Roots: none

y-intercept: none

Increasing intervals: none

Decreasing intervals: $(-\infty,0) \cup (0,\infty)$

Relative max/min: none

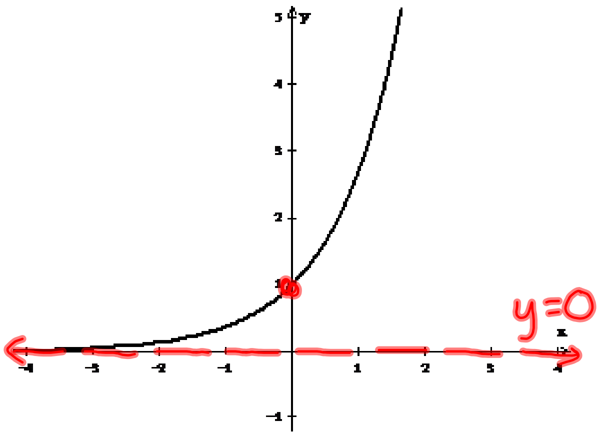
Continuous?: no Even/odd? : odd

* Other: vertical $x=0$, horizontal $y=0$ Bounded?: no

End behavior: $\lim_{x\to+\infty} f(x)=0$ $\lim_{x\to-\infty} f(x)=0$

* asymptotes are infinite discontinuities

The Exponential Function



$$f(x) = e^x$$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Roots: none

y-intercept: $(0, 1)$

Increasing intervals: $(-\infty, \infty)$

Decreasing intervals: none

Relative max/min: none

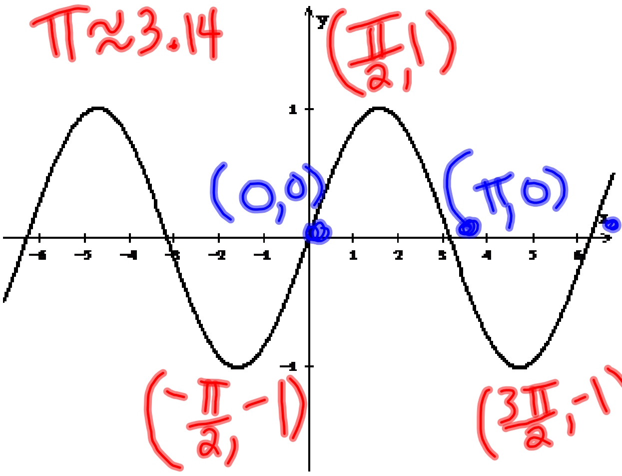
Continuous?: yes Even/odd? : neither

Other: Horizontal Asymptote $y=0$ Bounded?: below

End behavior: $\lim_{x \to +\infty} f(x) = +\infty$ $\lim_{x \to -\infty} f(x) = 0$

The Sine Function

$\pi \approx 3.14$



$f(x) = \sin x$

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Roots: $x = \pi n$ $\quad n \in$ integer

y-intercept: $(0,0)$

Increasing intervals: $[-\pi/2, \pi/2]$ repeat every 2π cycl

Decreasing intervals: $[\pi/2, 3\pi/2]$ repeat every 2π cycles

Relative max/min: abs max @ $x = \frac{\pi}{2} + 2\pi n$ abs min @ $x = \frac{3\pi}{2} + 2\pi n$

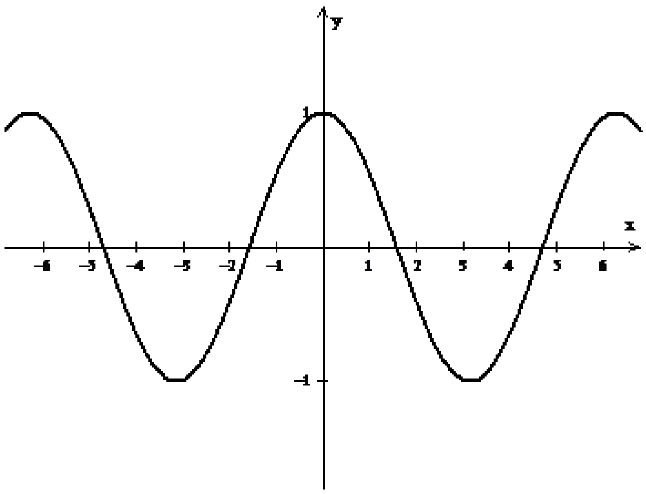
Continuous?: yes Even/odd? : odd

Other: periodic Bounded?: above and below

End behavior: oscillates between -1 and 1

increasing between $[-\pi/2, \pi/2)$ repeats every cycle.

The Cosine Function



$$f(x) = \cos x$$

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Roots: $x = \frac{\pi}{2}n$ n = odd integer

y-intercept: $(0, 1)$

Increasing intervals: $[\pi + 2\pi n,\ 2\pi + 2\pi n]$

Decreasing intervals: $[2\pi n,\ \pi + 2\pi n]$

Relative max/min: abs max @ $x = 0 + 2\pi n$ abs min @ $x = \pi + 2\pi n$

Continuous?: yes

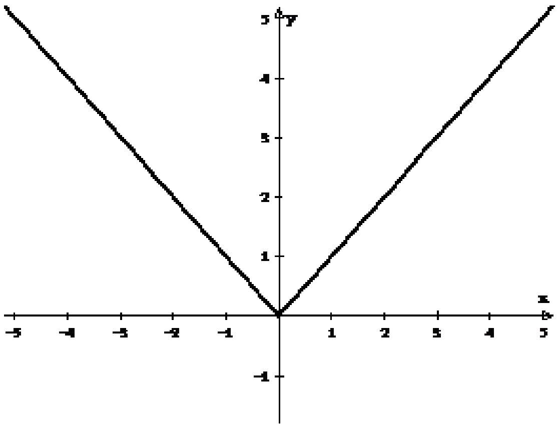
Even/odd? : even

Other: periodic

Bounded?: above and below

End behavior: oscillates between -1 and 1

The Absolute Value Function



$$f(x) = |x|$$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Roots: $x = 0$

y-intercept: $(0, 0)$

Increasing intervals: $[0, \infty)$

Decreasing intervals: $(-\infty, 0]$

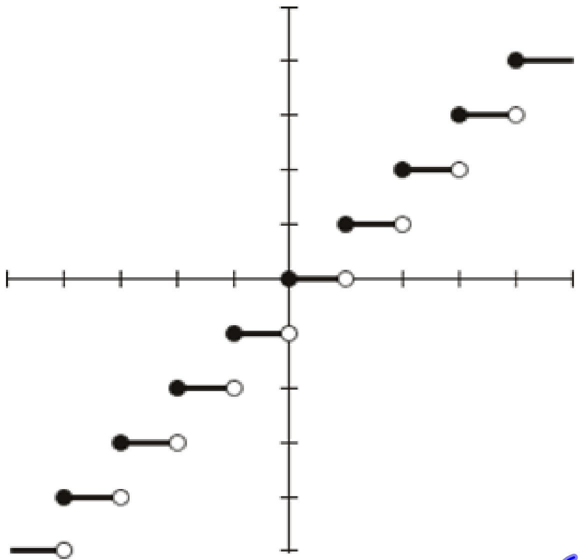
Relative max/min: absolute minimum @ $(0,0)$

Continuous?: yes Even/odd? : even

Other: rises left + right sharp v-shape Bounded?: below

End behavior: $\lim_{x \to +\infty} f(x) = +\infty$ $\lim_{x \to -\infty} f(x) = +\infty$

The Greatest Integer Function



$$f(x) = [x]$$

Domain: $(-\infty, \infty)$

Range: integers

Roots: $[0, 1)$

y-intercept: $(0, 0)$

Increasing intervals: $(-\infty, \infty)$

Decreasing intervals: none constant: $[n, n+1)$

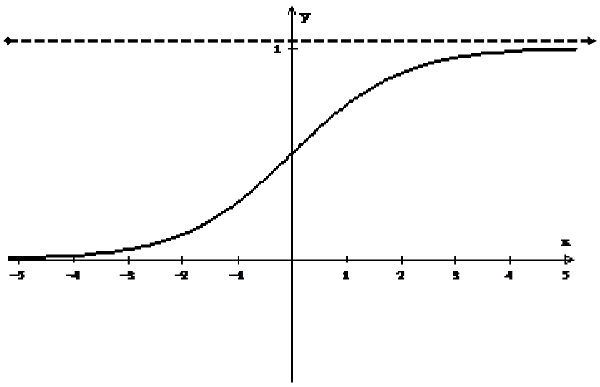
Relative max/min: none

Continuous?: no Even/odd? : neither

Other: jump discontinuities at integer values of x Bounded?: no

End behavior: $\lim_{x \to +\infty} f(x) = +\infty$ $\lim_{x \to -\infty} f(x) = -\infty$

The Logistic Function



$$f(x) = \frac{1}{1+e^{-x}}$$

Domain: $(-\infty, \infty)$

Range: $(0, 1)$

Roots: none

y-intercept: $(0, \frac{1}{2})$

Increasing intervals: $(-\infty, \infty)$

Decreasing intervals: none

Relative max/min: none

Continuous?: yes Even/odd? : neither

Other: two horizontal asymptotes Bounded?: above + below

End behavior: $\lim_{x \to +\infty} f(x) = 1$ $\lim_{x \to -\infty} f(x) = 0$