TWELVE BASIC FUNCTIONS

In advanced mathematics you will find it helpful to analyze functions that appear repeatedly. This lesson will help you recognize basic properties and characteristics of common functions.

DIRECTIONS
Give a complete analysis for each of the twelve basic functions. The analysis should include as many of the following as possible:

- Domain
- Range
- Continuity
- Increasing/decreasing behavior
- Symmetry
- Boundedness
- Local extrema
- Horizontal asymptotes
- Vertical asymptotes
- End behavior
- x-intercepts
- y-intercepts
The Identity Function

\[ f(x) = x \]

Domain: \((\mathbb{R})\)

Range: \((\mathbb{R})\)

Roots: \(x = 0\)

\(y\)-intercept: \((0,0)\)

Increasing intervals: \((\mathbb{R})\)

Decreasing intervals: none

Relative max/min: none

Continuous?: yes

Even/odd?: odd

Other: rises right, falls left

Bounded?: not

End behavior:
\[
\lim_{{x \to +\infty}} f(x) = +\infty, \quad \lim_{{x \to -\infty}} f(x) = -\infty
\]
The Squaring Function

\[ f(x) = x^2 \]

Domain: \((-\infty, \infty)\)

Range: \([0, \infty)\)

Roots: \(x = 0\)

\(y\)-intercept: \((0,0)\)

Increasing intervals: \([0, \infty)\)

Decreasing intervals: \([-\infty, 0]\)

Relative max/min: absolute min @ \((0,0)\)

Continuous?: yes

Even/odd?: even

Other: rises on left and right

Bounded?: below

End behavior:
\[ \lim_{x \to +\infty} f(x) = +\infty \]
\[ \lim_{x \to -\infty} f(x) = +\infty \]
The Cubing Function

\[ f(x) = x^3 \]

Domain: \((-\infty, \infty)\)
Range: \((-\infty, \infty)\)
Roots: \(x = 0\)
y-intercept: \((0,0)\)

Increasing intervals: \((-\infty, \infty)\)
Decreasing intervals: none
Relative max/min: none
Continuous?: yes Even/odd?: odd
Other: \(\text{falls left, rises right}\) Bounded?: no
End behavior: \(\lim_{x \to +\infty} f(x) = +\infty\) \(\lim_{x \to -\infty} f(x) = -\infty\)
The Square Root Function

\[ f(x) = \sqrt{x} \]

*Domain:* \([0, \infty)\)

*Range:* \([0, \infty)\)

*Roots:* \(x = 0\)

*\(y\)-intercept:* \((0,0)\)

*Increasing intervals:* \([0, \infty)\)

*Decreasing intervals:* none

*Relative max/min:* absolute minimum \@ \((0,0)\)

*Continuous?:* yes  Even/odd?: neither

*Other:*  Bounded?: below

*End behavior:* \(\lim_{x \to +\infty} f(x) = +\infty\)  \(\lim_{x \to -\infty} f(x) = D.N.E.\)

\(D.N.E.\) means does not exist
The Natural Logarithm Function

\[ f(x) = \ln x \]

Domain: \((0, \infty)\)

Range: \((-\infty, \infty)\)

Roots: \(x = 1\)

\(y\)-intercept: \(\text{none}\)

Increasing intervals: \((0, \infty)\)

Decreasing intervals: \(\text{none}\)

Relative max/min: \(\text{none}\)

Continuous?: \(\text{yes}\) Even/odd?: \(\text{neither}\)

Other: \(x = 0\) Bounded?: \(\text{no}\)

End behavior:
\[
\lim_{x \to +\infty} f(x) = +\infty \quad \lim_{x \to -\infty} f(x) = \text{D.N.E.}
\]
The Reciprocal Function

\[ f(x) = \frac{1}{x} \]

Domain: \((-\infty, 0) \cup (0, \infty)\)

Range: \((-\infty, 0) \cup (0, \infty)\)

Roots: \text{none}

\(y\)-intercept: \text{none}

Increasing intervals: \text{none}

Decreasing intervals: \((-\infty, 0) \cup (0, \infty)\)

Relative max/min: \text{none}

Continuous?: \text{no} \quad \text{Even/odd?} : \text{odd}

Other: \text{vertical} \quad x=0 \quad \text{horizontal} \quad y=0

Bounded?: \text{no}

End behavior: \(\lim_{x \to +\infty} f(x) = 0\) \quad \(\lim_{x \to -\infty} f(x) = 0\)

*asymptotes are infinite discontinuities
The Exponential Function

\[ f(x) = e^x \]

Domain: \((-\infty, \infty)\)
Range: \((0, \infty)\)
Roots: none
\(y\)-intercept: \((0,1)\)
Increasing intervals: \((-\infty, \infty)\)
Decreasing intervals: none
Relative max/min: none
Continuous?: yes Even/odd? : neither
Other: Horizontal Asymptote \(y = 0\) Bounded?: below
End behavior: \[ \lim_{x \to +\infty} f(x) = +\infty \] \[ \lim_{x \to -\infty} f(x) = 0 \]
The Sine Function

\[ f(x) = \sin x \]

Domain: \((-\infty, \infty)\)
Range: \([-1, 1]\)
Roots: \(x = \pi n\) \(n\) \(\epsilon\) integer
\(y\)-intercept: \((0, 0)\)
Increasing intervals: \([-\pi/2, \pi/2]\) repeat every \(2\pi\) cycles
Decreasing intervals: \([\pi/2, 3\pi/2]\) repeat every \(2\pi\) cycles,
Relative max/min: \(\text{max} \ L_x = \frac{\pi}{2} + 2\pi n\) \(\text{min} \ R_x = \frac{3\pi}{2} + 2\pi n\)
Continuous?: \(\text{yes}\) Even/odd?: \(\text{odd}\)
Other: \(\text{periodic}\) Bounded?: \(\text{above and below}\)
End behavior: \(\text{oscillates between } -1 \text{ and } 1\)

Increasing between \([-\pi/2, \pi/2]\) repeats every cycle.
The Cosine Function

\[ f(x) = \cos x \]

Domain: \((-\infty, \infty)\)
Range: \([-1, 1]\)
Roots: \(x = \frac{\pi}{2n} n=\text{odd}\) \(\text{integer}\)
y-intercept: \((0,1)\)
Increasing intervals: \([\pi + 2\pi n, \pi + 2\pi n]\)
Decreasing intervals: \([2\pi n, \pi + 2\pi n]\)
Relative max/min: 
Continuous?: yes  Even/odd?: even
Other: periodic  Bounded?: below
End behavior: oscillates between -1 and 1
The Absolute Value Function

\[ f(x) = |x| \]

Domain: \((-\infty, \infty)\)

Range: \([0, \infty)\)

Roots: \(x = 0\)

\(y\)-intercept: \((0,0)\)

Increasing intervals: \([0, \infty)\)

Decreasing intervals: \((-\infty, 0]\)

Relative max/min: \(\text{absolute minimum} @ (0,0)\)

Continuous?: yes Even/odd?: even

Other: rises left & right sharp v-shape

Bounded?: below

End behavior: \(\lim_{x \to +\infty} f(x) = +\infty\) \(\lim_{x \to -\infty} f(x) = +\infty\)
The Greatest Integer Function

\[ f(x) = [x] \]

Domain: \((-\infty, \infty)\)

Range: integers

Roots: \([0, 1)\)

\(y\)-intercept: \((0, 0)\)

Increasing intervals: \((-\infty, \infty)\)

Decreasing intervals: none, constant: \([n, n+1)\)

Relative max/min: none

Continuous?: no, Even/odd?: neither

Jump discontinuities at integer values of \(x\)

Bounded?: no

End behavior:
\[
\lim_{{x \to +\infty}} f(x) = +\infty, \quad \lim_{{x \to -\infty}} f(x) = -\infty
\]
The Logistic Function

\[ f(x) = \frac{1}{1 + e^{-x}} \]

Domain: \((-\infty, \infty)\)

Range: \((0, 1)\)

Roots: none

\(y\)-intercept: \((0, \frac{1}{2})\)

Increasing intervals: \((-\infty, \infty)\)

Decreasing intervals: none

Relative max/min: none

Continuous?: yes Even/odd?: neither

Other: two horizontal asymptotes

Bounded?: above + below

End behavior: \(\lim_{x \to \infty} f(x) = 1\) \(\lim_{x \to -\infty} f(x) = 0\)