5-4 Dividing Polynomials

Vocabulary

Review

1. Circle the *factors* of $x^3 - 4x^2$.

x^2	x^3	$x - 4$	$-4x^2$

2. Cross out the expression that is NOT a *factor* of $2x^5 + 8x^3$.

x^2	$2x^3$	$2x^2 + 8$	$8x^3$

Vocabulary Builder

6 ⟵ **quotient**
divisor ⟶ $3\overline{)19}$ ⟵ dividend
18
1 ⟵ remainder

quotient (noun) KWOH shunt

Related Words: dividend, divisor, remainder

Main Idea: A **quotient** is the simplification of a division expression.

Use Your Vocabulary

3. Circle the dividend and underline the divisor in each *quotient*.

$\frac{x}{5}$	$3.2 \div 16$	$15\overline{)100}$	two divided by seven

Problem 1 Polynomial Long Division

Got It? Use polynomial long division to divide $3x^2 - 29x + 56$ by $x - 7$. What are the quotient and remainder?

4. Use the justifications to divide the expressions.

$$\begin{array}{r} 3x - \square \\ x - 7\overline{)3x^2 - 29x + 56} \\ -(3x^2 - \square x) \\ \hline -8x + 56 \\ \square + \square \\ \hline \square \end{array}$$

Divide the first term in the dividend by the first term in the divisor to get the first term in the quotient: $3x^2 \div x = 3x$.

Multiply the first term in the quotient by the divisor: $3x(x - 7)$.

Subtract to get $-8x$. Bring down 56.

Divide $-8x$ by x.

Subtract to find the remainder.

5. Identify each part of the problem.

Dividend

Divisor

Quotient

Remainder

6. Check your solution.

take note

Key Concept The Division Algorithm for Polynomials

You can divide polynomial $P(x)$ by polynomial $D(x)$ to get polynomial quotient $Q(x)$ and polynomial remainder $P(x)$. The result is $P(x) = D(x)Q(x) + R(x)$.

$$\begin{array}{r} Q(x) \\ D(x)\overline{)P(x)} \\ * \\ * \\ * \\ \hline R(x) \end{array}$$

If $R(x) = 0$, then $P(x) = D(x)Q(x)$ and $D(x)$ and $Q(x)$ are factors of $P(x)$.

To use long division, $P(x)$ and $D(x)$ should be in standard form with zero coefficients where appropriate. The process stops when the degree of the remainder, $P(x)$, is less than the degree of the divisor, $D(x)$.

7. Cross out the polynomials that are NOT in the correct form for long division.

$x^3 - 7x + 2$ $\quad 2x^4 + 3x$ $\quad 4x^3 + 9x^2 + 0x - 12$

Problem 2 Checking Factors

Got It? **Is $x^4 - 1$ a factor of $P(x) = x^5 + 5x^4 - x - 5$? If it is, write $P(x)$ as a product of two factors.**

8. Divide.

$$x^4 - 1\overline{)x^5 + 5x^4 + 0x^3 + 0x^2 - x - 5}$$

9. Write $P(x)$ as a product of two factors.

Underline the correct word(s), number, or expression to complete each sentence.

10. The remainder of the quotient is 0 / $x + 5$ / $x - 5$.

11. The expression $x^4 - 1$ is / is not a factor of $P(x) = x^5 + 5x^4 - x - 5$.

Problem 3 Using Synthetic Division

Got It? **Use synthetic division to divide $x^3 - 57x + 56$ by $x - 7$. What are the quotient and remainder?**

12. Do the synthetic division. Remember that the sign of the number in the divisor is reversed.

7	1	0			Write the coefficients of the polynomial.
					Bring down the first coefficient. Multiply the coefficient by the divisor.
					Add to the next coefficient. Continue multiplying and adding through the last coefficient.

13. The quotient is ______, and the remainder is ______.

Problem 4 Using Synthetic Division to Solve a Problem

Got It? **Crafts** **If the polynomial $x^3 + 6x^2 + 11x + 6$ expresses the volume, in cubic inches, of a shadow box, and the width is $(x + 1)$ in., what are the dimensions of the box?**

14. Use synthetic division.

15. Factor the quotient.

16. The height of the box is ______ in., the width of the box is ______ in., and the length of the box is ______ in.

take note

Theorem The Remainder Theorem

If you divide a polynomial $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is $P(a)$.

17. If you divide $3x^2 + x - 5$ by $x - 1$, the remainder is $P(\quad)$.

18. If you divide $2x^2 + x + 6$ by $x + 1$, the remainder is $P(\quad)$.

Problem 5 Evaluating a Polynomial

Got It? **What is $P(-4)$, given $P(x) = x^5 - 3x^4 - 28x^3 + 5x + 20$?**

19. $P(-4)$ is the remainder when you divide

$x^5 - 3x^4 - 28x^3 + 5x + 20$ by $x - 4$ / $4 - x$ / $x + 4$

20. Use synthetic division. Circle the remainder.

21. $P(-4) =$ ____

Lesson Check • Do you UNDERSTAND?

Reasoning **A polynomial $P(x)$ is divided by a binomial $x - a$. The remainder is zero. What conclusion can you draw? Explain.**

Write T for *true* or F for *false*.

____ **22.** One factor of the polynomial is $x - a$.

____ **23.** One root of the polynomial is $-a$.

____ **24.** An x-intercept of the graph of $y = P(x)$ is a.

25. If $P(x)$ is divided by $x - a$ then $P(a) =$ the remainder and $P(x) = (x - a)(Q(x))$.
This illustrates the Division Algorithm / Remainder Theorem / Factor Theorem .

26. If the remainder of $P(x)$ divided by $x - a$ is zero, what do you know about the factors and roots of $P(x)$?

____

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Math Success

Check off the vocabulary words that you understand.

☐ polynomial ☐ synthetic division ☐ Remainder Theorem

Rate how well you can *divide polynomials*.

Need to review 0 2 4 6 8 10 Now I get it!