Precalculus 2nd Semester Exam Review

Use basic identities to simplify the expression.

1) $\dfrac{\tan\theta}{\sec\theta}$

      A) $\tan^2\theta$ B) $\cos^3\theta$ C) $\sin\theta$ D) $\sec^2\theta$

1) _____

2) $\dfrac{\csc\theta\cot\theta}{\sec\theta}$

      A) $\cot^2\theta$ B) 1 C) $\sec^2\theta$ D) $\csc^2\theta$

2) _____

3) $\cos\theta - \cos\theta\sin^2\theta$

      A) $\tan^2\theta$ B) $\sec^2\theta$ C) $\cos^3\theta$ D) $\sin\theta$

3) _____

Simplify the expression.

4) $\dfrac{\sin^2 x - 1}{\cos(-x)}$

      A) $\cos x$ B) $-\cos x$ C) $-\sin x$ D) $\sin x$

4) _____

Find an exact value.

5) $\sin 15°$

      A) $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$ B) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ C) $\dfrac{-\sqrt{6}+\sqrt{2}}{4}$ D) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

5) _____

6) $\cos 75°$

      A) $-\sqrt{6}+\sqrt{2}$ B) $\sqrt{6}-\sqrt{2}$ C) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ D) $\dfrac{-\sqrt{6}+\sqrt{2}}{4}$

6) _____

7) $\cos\dfrac{\pi}{12}$

      A) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ B) $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$ C) $\dfrac{-\sqrt{6}+\sqrt{2}}{4}$ D) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

7) _____

8) $\sin\dfrac{-11\pi}{12}$

      A) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ B) $\dfrac{\sqrt{2}-\sqrt{6}}{4}$ C) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ D) $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$

8) _____

Are $\overrightarrow{RS}$ and $\overrightarrow{OP}$ equivalent?

9) R = (8,   4), S = (15,   13), O = (4,   9), and P = (11,   18)

      A) No   B) Yes

9) _____

Prove that $\overrightarrow{RS}$ and $\overrightarrow{OP}$ are equivalent by showing that they represent the same vector.

10) R = (-1,   -5), S = (0,   -8), O = (0, 0), and P = (1,   -13)

      A) Yes   B) No

10) _____

Find the component form of the indicated vector.

11) Let $\mathbf{u} = \langle -1, 5\rangle$, $\mathbf{v} = \langle -2, -1\rangle$. Find $-\mathbf{u} - \mathbf{v}$.

      A) $\langle -3, 4\rangle$ B) $\langle -1, -6\rangle$ C) $\langle 1, 6\rangle$ D) $\langle 3, -4\rangle$

11) _____

12) Let $\mathbf{u} = \langle -2, 7 \rangle$, $\mathbf{v} = \langle -6, -5 \rangle$. Find $\mathbf{v} - \mathbf{u}$.

    A) $\langle -4, -12 \rangle$ B) $\langle -3, -13 \rangle$ C) $\langle 9, 1 \rangle$ D) $\langle -8, 2 \rangle$

12) _____

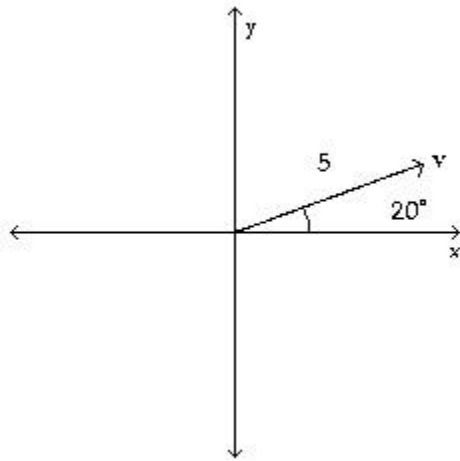**Find the component form and magnitude of the indicated vector.**

13) Given that P = (8,   6) and Q = (7,   15), find the component form and magnitude of the vector $\overrightarrow{PQ}$.

    A) $\langle 1, -9 \rangle$, $\sqrt{82}$ B) $\langle -1, 9 \rangle$, $\sqrt{82}$ C) $\langle -1, 9 \rangle$, 82   D) $\langle 1, -9 \rangle$, 82

13) _____

Find the component form of the indicated vector.

14) Let $\mathbf{u} = \langle 5, -4 \rangle$, $\mathbf{v} = \langle -6, 6 \rangle$. Find $-3\mathbf{u} + 5\mathbf{v}$.

    A) $\langle -45, 42 \rangle$ B) $\langle 15, -18 \rangle$ C) $\langle 3, -6 \rangle$ D) $\langle -45, 2 \rangle$

14) _____

Find the component form of the vector v.

15)

    A) $\approx \langle 4.70, 1.71 \rangle$ B) $\approx \langle 1.71, 4.70 \rangle$ C) $\approx \langle 4.70, 1.82 \rangle$ D) $\approx \langle 0.94, 0.34 \rangle$

15) _____

Find the unit vector in the direction of the given vector. Write your answer in the indicated form.

16) Let $\mathbf{u} = \langle -2, -3 \rangle$. Find the unit vector in the direction of $\mathbf{u}$, and write your answer as a linear combination of the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$.

    A) $\frac{-2}{13}\mathbf{i} + \frac{-3}{13}\mathbf{j}$ B) $\frac{-2}{\sqrt{13}}\mathbf{i} + \frac{-3}{\sqrt{13}}\mathbf{j}$ C) $\frac{-2}{5}\mathbf{i} + \frac{-3}{5}\mathbf{j}$ D) $\mathbf{i} + \mathbf{j}$

16) _____

17) Let $\mathbf{u} = \langle -1, -3 \rangle$. Find the unit vector in the direction of $\mathbf{u}$, and write your answer in component form.

    A) $\langle 1, 1 \rangle$ B) $\left\langle -\frac{1}{10}, \frac{-3}{10} \right\rangle$ C) $\left\langle -\frac{1}{4}, \frac{-3}{4} \right\rangle$ D) $\left\langle \frac{-1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right\rangle$

17) _____

Find the magnitude and direction angle for the following vector. Give the direction angle as an angle in [0°, 360°) rounded to the nearest tenth.

18) $10(\cos 217°\mathbf{i} + \sin 217°\mathbf{j})$

    A) 100, 37°   B) $\sqrt{10}$, 217°   C) 10, 37°   D) 10, 217°

18) _____

19) $\langle -13, 13\sqrt{3} \rangle$ 19) _____
    A) 26, 240°   B) 39, 240°   C) 26, 120°   D) 39, 300°

Find a· b.

20) $a = \langle 5, -1 \rangle$, $b = \langle 9, 2 \rangle$ 20) _____
    A) 47   B) $\langle 45, -2 \rangle$ C) $\langle 14, 1 \rangle$ D) 43

21) $a = 7i - 4j$, $b = 4i + 3j$ 21) _____
    A) $\langle 28, -12 \rangle$ B) $\langle 11, -1 \rangle$ C) 16   D) –40

Find the angle between the given vectors to the nearest tenth of a degree.

22) $u = \langle 4, -4 \rangle$, $v = \langle -6, -6 \rangle$ 22) _____
    A) 180°   B) 90°   C) 0°   D) 45°

23) $u = \sqrt{5}i - 9j$, $v = \sqrt{5}i + j$ 23) _____
    A) 65.9°   B) 100.1°   C) 90.4°   D) 45.2°

Given the polar coordinate (–4, –π/3), find the rectangular coordinate.

24)   24) _____
    A) $(2, 2\sqrt{3})$ B) $(-2, 2\sqrt{3})$ C) $(2, -2\sqrt{3})$ D) $(-2, -2\sqrt{3})$

Determine two pairs of polar coordinates for the rectangular point (0, $\sqrt{3}$). Assume 0° ≤ θ < 360°.

25)   25) _____
    A) $(\sqrt{3}, 90°), (-\sqrt{3}, 270°)$ B) $(\sqrt{3}, 270°), (-\sqrt{3}, 90°)$
    C) (3, 90°), (–3, 270°)   D) (3, 0°), (–3, 180°)

Find the rectangular coordinates of the point with the given polar coordinates.

26) (–7, 2π/3)   26) _____
    A) $\left[\dfrac{7}{2}, \dfrac{-7}{2}\right]$ B) $\left[\dfrac{-7}{2}, \dfrac{7}{2}\right]$ C) $\left[\dfrac{-7\sqrt{3}}{2}, \dfrac{7}{2}\right]$ D) $\left[\dfrac{7}{2}, \dfrac{-7\sqrt{3}}{2}\right]$

Determine two pairs of polar coordinates for the point with 0° ≤ θ < 360°.

27) $\left[-\dfrac{5\sqrt{3}}{2}, \dfrac{5}{2}\right]$ 27) _____
    A) (5, 300°), (–5, 120°)   B) (5, 30°), (–5, 210°)
    C) (5, 150°), (–5, 330°)   D) (5, 240°), (–5, 60°)

Find an equivalent equation in rectangular coordinates.

28) r sin θ = 10   28) _____
    A) y = 10x   B) x = 10   C) y = 10   D) x = 10y

Find an equivalent equation in polar coordinates.

29) 2x + 3y = 6   29) _____
    A) 2 sin θ + 3 cos θ = 6r   B) r(2 cos θ + 3 sin θ) = 6
    C) r(2 sin θ + 3 cos θ) = 6   D) 2 cos θ + 3 sin θ = 6r

Solve the problem.

30) Find the equilibrium point for the given demand and supply curve.

$p = 394 - 2x$ (demand)

$p = 3x - 261$ (supply)

   A) (131, 132)   B) (−261, 394)   C) (132, 131)   D) (392, 264)

30) _____

31) In one study the maximum heart rates of conditioned athletes were examined. A group of athletes was exercised to exhaustion. Let x represent an athlete's heart rate five seconds after stopping exercise and y represent an athlete's heart rate ten seconds after stopping exercise. It was found that the maximum heart rate H for these athletes satisfied the following two equations.

$H = 0.491 x + 0.468 y + 11.2$

$H = -0.981 x + 1.872y + 26.4$

If an athlete had a maximum heart rate of H = 151, determine the value of x graphically to the tenths place.

   A) x = 147.1   B) x = 148.2   C) x = 147.9   D) x = 147.6

31) _____

Find the indicated matrix.

32) Let $C = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$ and $D = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$. Find $C - 4D$.

A) $\begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}$ B) $\begin{bmatrix} -5 \\ 15 \\ 6 \end{bmatrix}$ C) $\begin{bmatrix} 5 \\ -6 \\ 4 \end{bmatrix}$ D) $\begin{bmatrix} 5 \\ -15 \\ -6 \end{bmatrix}$

32) _____

Determine the value of each variable.

33) $\begin{bmatrix} 6 & 9 & -6 \\ 1 & m & -2 \end{bmatrix} = \begin{bmatrix} x & y & -6 \\ 1 & 7 & -2 \end{bmatrix}$

   A) x = −6; y = −9; m = −7   B) x = −6; y = 9; m = −7

   C) x = 9; y = 6; m = 7   D) x = 6; y = 9; m = 7

33) _____

Find the indicated matrix product or state that the product is undefined.

34) $A = \begin{bmatrix} 8 \\ -5 \end{bmatrix}, B = \begin{bmatrix} -5 & 8 \\ 4 & -7 \end{bmatrix}$

AB

A) $\begin{bmatrix} -60 & 99 \end{bmatrix}$ B) $\begin{bmatrix} -40 & 64 \\ -20 & 35 \end{bmatrix}$ C) $\begin{bmatrix} -80 \\ 67 \end{bmatrix}$ D) Undefined

34) _____

Find the inverse of A if it has one, or state that the inverse does not exist.

35)
$$A = \begin{bmatrix} 1 & 0 \\ 3 & -6 \end{bmatrix}$$

35) _____

A)
$$\begin{bmatrix} 1 & 0 \\ \dfrac{1}{2} & -\dfrac{1}{6} \end{bmatrix}$$

B) Inverse does not exist

C)
$$\begin{bmatrix} -\dfrac{1}{6} & 0 \\ \dfrac{1}{2} & 1 \end{bmatrix}$$

D)
$$\begin{bmatrix} 1 & 0 \\ -\dfrac{1}{2} & -\dfrac{1}{6} \end{bmatrix}$$

Solve the system of equations by using an inverse matrix.

36) $-2x + 9y - z = -5$
$\quad x - 8y + 9z = 55$
$\quad 6x + y + z = 57$

36) _____

A) $(8, 7, 2)$ B) $\varnothing$ C) $(-8, 2, 16)$ D) $(8, 2, 7)$

Solve the problem.

37)

37) _____
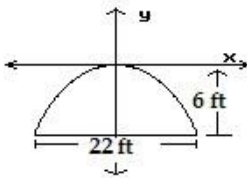
A building has an entry the shape of a parabolic arch 6 ft high and 22 ft wide at the base. Find an equation for the parabola if the vertex is put at the origin of the coordinate system.

A) $x^2 = -80.7y$ B) $x^2 = -20.2y$ C) $y^2 = -80.7x$ D) $y^2 = -20.2x$

Find the vertex, focus, directrix, and focal width of the parabola.

38) $x = 8y^2$

38) _____

A) Vertex: $(0, 0)$; Focus: $\left(\dfrac{1}{32}, 0\right)$; Directrix: $x = -\dfrac{1}{32}$; Focal width: 0.13

B) Vertex: $(0, 0)$; Focus: $\left(0, \dfrac{1}{32}\right)$; Directrix: $y = -\dfrac{1}{32}$; Focal width: 32

C) Vertex: $(0, 0)$; Focus: $\left(\dfrac{1}{32}, 0\right)$; Directrix: $x = \dfrac{1}{32}$; Focal width: 32

D) Vertex: $(0, 0)$; Focus: $\left(\dfrac{1}{8}, 0\right)$; Directrix: $x = -\dfrac{1}{8}$; Focal width: 0.13

Find the center, vertices, and foci of the ellipse with the given equation.

39) $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

39) _____

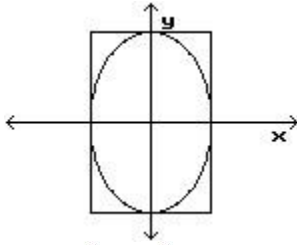A) Center: $(0, 0)$; Vertices: $(-5, 0), (5, 0)$; Foci: $(-4, 0), (4, 0)$
B) Center: $(0, 0)$; Vertices: $(-5, 0), (5, 0)$; Foci: $(-3, 0), (3, 0)$
C) Center: $(0, 0)$; Vertices: $(0, -5), (0, 5)$; Foci: $(0, -3), (0, 3)$
D) Center: $(0, 0)$; Vertices: $(0, -5), (0, 5)$; Foci: $(0, -4), (0, 4)$

Solve the problem.

40) An elliptical riding path is to be built on a rectangular piece of property that measures 10 mi by 6 mi. Find an equation for the ellipse if the path is to touch the center of the property line on all 4 sides

40) _____

A) $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$

B) $\dfrac{x^2}{100} + \dfrac{y^2}{9} = 1$

C) $\dfrac{x^2}{25} + \dfrac{y^2}{6} = 1$

D) $\dfrac{x^2}{9} + \dfrac{y^2}{100} = 1$

41) A comet follows the hyperbolic path described by $\dfrac{x^2}{21} - \dfrac{y^2}{18} = 1$, where x and y are in millions. If the sun is the focus of the path, how close to the sun is the vertex of the path?

41) _____
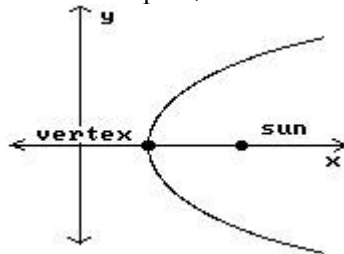
A) 4.6 million   B) 39 million   C) 1.6 million   D) 6.2 million

Write out the first five terms of the sequence.

42) $c_n = \dfrac{n+2}{3}$

42) _____

A) $\dfrac{2}{3}, \dfrac{2}{3}, \dfrac{2}{3}, \dfrac{2}{3}, \dfrac{2}{3}$

B) $1, \dfrac{4}{3}, \dfrac{5}{3}, 2, \dfrac{7}{3}$

C) $\dfrac{2}{3}, 1, \dfrac{4}{3}, \dfrac{5}{3}, 2$

D) $1\dfrac{2}{3}, 2\dfrac{2}{3}, 3\dfrac{2}{3}, 4\dfrac{2}{3}, 5\dfrac{2}{3}$

Find the first six terms of the sequence.

43) $a_1 = -3, a_n = 5 \cdot a_{n-1}$

43) _____

A) −3, −15, −75, −375, −1875, −9375

B) 0, 5, −15, −10, −5, 0

C) −15, −75, −375, −1875, −9375, −46,875

D) −3, −15, −10, −5, 0, 5

Find an explicit rule for the nth term of the arithmetic sequence.

44) −14, −20, −26, −32, ...

44) _____

A) $a_n = -14 + (-6)(n+1)$

B) $a_n = -14 + (-6)(n-1)$

C) $a_n = -14 - (-6)(n+1)$

D) $a_n = -14 \times (-6)(n-1)$

Find an explicit rule for the nth term of the sequence.

45) 2, −2, 2, −2, ...

    A) $a_n = 2 \cdot (-1)^n$ B) $a_n = 2 \cdot (-1)^{n-1}$ C) $a_n = 2 \cdot 1^{n+1}$ D) $a_n = 2 \cdot 1^{n-1}$

45) _____

Solve.

46) A collection of dimes is arranged in a triangular array with 18 coins in the base row, 17 in the next, 16 in the next, and so forth with 1 dime in the last row. Find the value of the collection.

    A) $ 1.71   B) $ 34.20   C) $ 8.55   D) $ 17.10

46) _____

Find the sum of the geometric sequence.

47) $\frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{16}{3}, \frac{32}{3}$

    A) $\frac{61}{15}$ B) $\frac{61}{3}$ C) $\frac{62}{3}$ D) $\frac{62}{15}$

47) _____

48) 2, −8, 32, −128, 512

    A) 682   B) 410   C) −410   D) −682

48) _____

Determine whether the infinite geometric series converges. If the series converges, determine the limit.

49) 6 + 24 + 96 + 384 + ...

    A) Converges; 2046   B) Converges; 510

    C) Diverges   D) Converges; 126

49) _____

Find the mean for the given sample data.

50) The students in Hugh Logan's math class took the Scholastic Aptitude Test. Their math scores are shown below. Find the mean score.

603 632 350 343 565
346 341 535 470 482

    A) 476.0   B) 457.5   C) 466.7   D) 476.2

50) _____

Identify whether the average described is a mean, median, or mode.

51) The physics teacher grades on a curve. She gives A's to students whose scores are in the top 10%, B's to those whose scores fall in the next 20%, Fs to those with scores in the bottom 10%, Ds to the next 20%, and those with average scores get a C.

    A) Mean   B) Median   C) Mode

51) _____

Find the mean. Round to the nearest tenth.

52)

Value	Frequency
124	2
187	3
222	4
307	2
358	3

    A) 85.6   B) 306.7   C) 241.8   D) 282.1

52) _____

Determine the five-number summary of the given data.

53) The weekly salaries (in dollars) of sixteen government workers are listed below.   53) _____

690	610	813	653
728	572	484	630
525	671	685	450
551	787	511	826

A) 450, 531.50, 641.5, 718.5, 826   B) 450, 531.50, 630, 718.5, 826
C) 450, 538.0, 641.5, 709, 826   D) 450, 525, 630, 690, 826

Solve the problem.

54) The test scores of 19 students are listed below. Find the interquartile range.   54) _____

91 49 86 71 59
62 97 56 90 79
82 83 51 88 72
42 92 94 66

A) 28.5   B) 30.5   C) 31.5   D) 31

Obtain the population standard deviation, σ, for the given data. Assume that the data represent population data. Round your final answer to one more decimal place than that used for the observations.

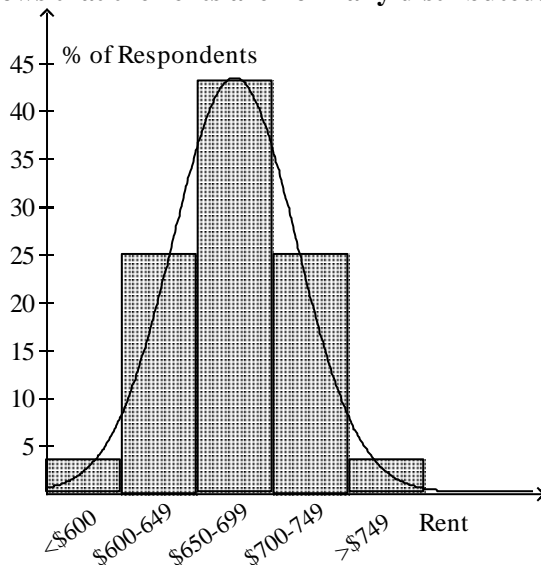55) The number of years of teaching experience is given below for 12 high-school teachers.   55) _____

22   17   15   30   12   19
17   8   5   13   22   31

A) 7.9   B) 9.5   C) 57.1   D) 7.6

Exam Rev

The bar graph shows the rents paid per month for apartments in an urban neighborhood. The curve shows that the rents are normally distributed.

____ 1. Estimate the percent of apartment residents who pay from $600 to $749 per month.

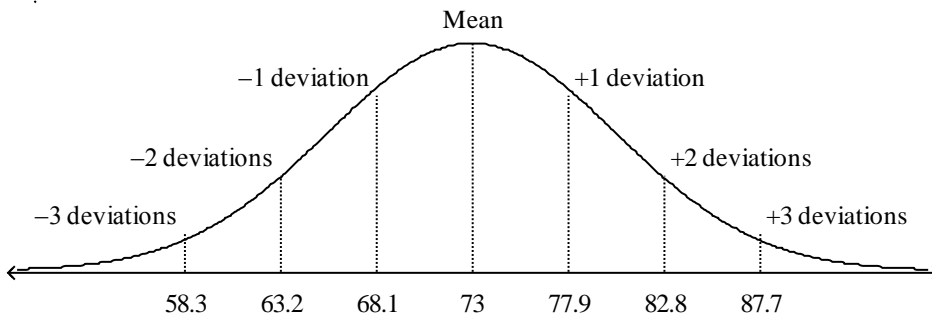A. 93%   B. 68%   C. 43%   D. 25%

_____ 2. Estimate the percent of apartment residents who pay less than $600 per month.

     A. 99%   B. 25%   C. 68%   D. 3%

_____ 3. The scores on an exam are normally distributed, with a mean of 74 and a standard deviation of 7. What percent of the scores are less than 81?

     A. 50%   B. 16%   C. 84%   D. 13.5%

_____ 4. A grocery store will only accept yellow onions that are at least 2.75 in. in diameter. A grower has a crop of onions with diameters that are normally distributed, with a mean diameter of 3.25 in. and a standard deviation of 0.25 in. What percent of the onions will be accepted by the grocery store?

     A. 16%   B. 2.5%   C. 97.5%   D. 34%

5. You took a test last week and the average grade was a 73.   The standard deviation was 4.9 points.   Sketch a normal curve labeling the horizontal axis at one, two, and three standard deviations from the mean.

1) C
2) A
3) C
4) B
5) D
6) C
7) A
8) B
9) B
10) B
11) D
12) A
13) A
14) A
15) A
16) B
17) D
18) D
19) C
20) D
21) C
22) B
23) B
24) B
25) A
26) D
27) C
28) C
29) B
30) A
31) D
32) D
33) D
34) D
35) A
36) D
37) B
38) A
39) B
40) A
41) C
42) B
43) A
44) B
45) B
46) D
47) C
48) B
49) C
50) C
51) B

52) C
53) C
54) D
55) D

E

1. ANS: A   REF: 11-9 Normal Distributions
2. ANS: D   REF: 11-9 Normal Distributions
3. ANS: C   REF: 11-9 Normal Distributions
4. ANS: C   REF: 11-9 Normal Distributions
5. ANS:

REF: 11-9 Normal Distributions