

CHAPTER 9

Sequences and Series

INTRO TO TODAY'S LESSON

N	Y
1	36
2	32
3	28
4	24
5	20
n	$40-4n$

- What is the sequence?
- Why must the input start at “1”?
- What is the domain of this sequence?
 - Why does this make sense?

DEFINITIONS

- ⦿ A sequence can always be graphed - the ordered pairs look like (n, a_n) , where n is the index (or position) and a_n is the term.
- ⦿ The *domain* of any sequence is Natural Numbers (also called counting numbers)
- ⦿ The graph is NOT continuous; it is a set of graphed points only
 - There is no term at the 2 and a half-th position

TWO TYPES OF RULES FOR SEQUENCES

○ Explicit

- Allows direct evaluation for any term given its position
- Works well for finding terms that are far apart
- Can be difficult to find

○ Recursive

- Requires an initial term to find the next term
- Defines each term using the first term as a starting point
- Easy to evaluate for sequential terms
- Works well for small portions of the sequence

DEFINITION: ARITHMETIC SEQUENCE

- A sequence with a pattern of addition or subtraction.
 - The difference between two consecutive numbers in the sequence will always be the same

RULES FOR ARITHMETIC SEQUENCES

○ Explicit:

- $a_n = a_1 + d(n - 1)$
 - a_n means any term in the sequence called “a”
 - a_1 is the first term
 - d is the pattern, called the constant difference
 - n is the index
- Can be simplified to $y = mx + b$

○ Recursive:

- $\begin{cases} a_1 = \text{the first term} \\ a_n = a_{n-1} + d \end{cases}$
 - Where d is the pattern (constant difference)
 - a_{n-1} is the notation that means the phrase “the previous term”

- Either rule can be written for a sequence, sometimes it's more useful to write a recursive. If you are not told which one to write, it is your choice.

IS THIS FORMULA RECURSIVE OR EXPLICIT?

- ⊙ $a_n = -4n^2 - 2$
- ⊙ $\begin{cases} a_1 = 3 \\ a_n = 2a_{n-1} + 3 \end{cases}$
- ⊙ $a_n = (n - 5)(n + 5)$
- ⊙ $a_n = -3a_{n-1}$, where $a_1 = -2$

WRITE A RULE FOR THIS SEQUENCE

⦿ 5, 8, 11, 14, ...

⦿ 21, 13, 5, -3

WRITE AN EXPLICIT RULE GIVEN THIS RECURSIVE FORMULA

⊙ $a_n = a_{n-1} + 4$ where $n = -9$

ERROR ANALYSIS

- ⦿ A student says the next term of the arithmetic sequence 0, 2, 4,... is 8. Explain and correct the students' error.

- ⦿ How could this be changed so the student's answer be correct?

ARITHMETIC MEAN

- To find a missing term between two other terms in an arithmetic sequence, find the arithmetic mean of the two given terms.
- For example, suppose you have 55, ____, 68.
To find the missing number, $\frac{55+68}{2} = 61.5$.
The sequence looks like 55, 61.5, 68