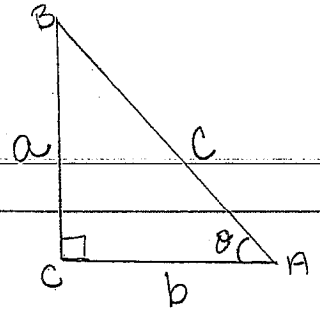
Pythagorean Identity: for any right $\triangle$ with $0° < \theta < 90°$, then

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

Why?



$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2}$$

add of fractions $\frac{a^2+b^2}{c^2}$ $\xrightarrow{\text{Pythag. Thm}}$ $\frac{c^2}{c^2} = 1$ ✓

Tangent Theorem: tangent can also be represented as: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

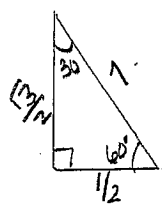
Why? $\tan\theta = \frac{a/c}{b/c}$

$$\frac{a}{c} \div \frac{b}{c}$$

$$\frac{a}{c} \cdot \frac{c}{b} = \frac{ac}{cb} = \frac{a}{b} = \frac{\text{opp leg } \theta}{\text{adj leg } \theta}$$ ✓

Two special right $\triangle$'s from geometry:

30°-60°-90°



$\sin 30° = \frac{1}{2}$

$\left(\frac{1}{2}\right)^2 + b^2 = 1^2$

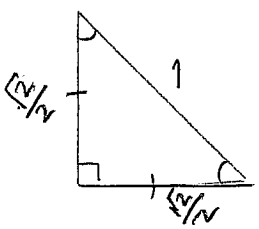
$\frac{1}{4} + b^2 = 1$

$b^2 = 3/4$

$b = \frac{\sqrt{3}}{\sqrt{4}}$

$b = \frac{\sqrt{3}}{2}$

45°-45°-90°



$x^2 + x^2 = 1^2$

$2x^2 = 1$

$x^2 = 1/2$

$x = \frac{\sqrt{1}}{\sqrt{2}}$

$x = \frac{1}{\sqrt{2}}$ Rationalize

$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$

These triangles are used heavily in trig because of their length relationships

θ	$\sin\theta$	$\cos\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

$\frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

$\frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

Properties of Trig Ratios

Alg II '11-'12

1. Use your calculator to evaluate:

i) $\sin 17° = .292$ iii) $\cos 25° = .906$

ii) $\cos 73° = .292$ iv) $\sin 65° = .906$

Find another pair of angle measures θ & β that illustrates the above pattern between i & ii and iii & iv

i) $\sin 13° = .225$ iii) $\sin 30° = .5$ $\cos 3°$

ii) $\cos 77° = .225$ iv) $\cos 60° = .5$ $\sin 87°$

Generalization: Suppose θ is between $0°$ and $90°$, then

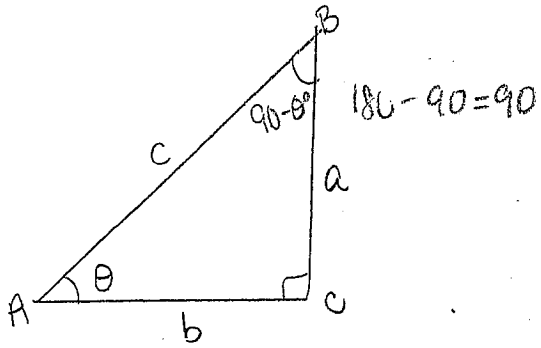
$\sin\theta = \cos(90-\theta)$ and $\cos\theta = \sin(90-\theta)$

These are called the Co-function Properties.

Show why these work using the right $\triangle$ to the right

$\sin\theta = \frac{a}{c}$ $\cos\theta = \frac{b}{c}$

$\cos(90-\theta) = \frac{a}{c}$ $\sin(90-\theta) = \frac{b}{c}$



Example:

$\sin 40° = \frac{\cos 50°}{(90-40)}$ $\cos 72° = \frac{\sin 18°}{(90-72)}$