# Precalculus 9.4.2 - Limits of Sequences

Definitions:

Finite sequence:

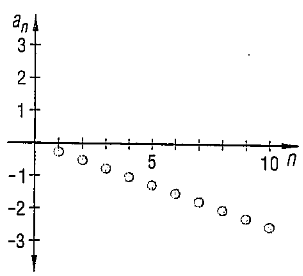
Infinite sequence:

End behavior:

Domain of a sequence:

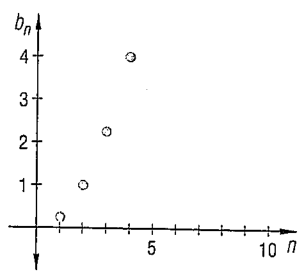
Consider the following four sequences and their graphs.

$a_n = -\frac{n}{4}$: $-\frac{1}{4}, -\frac{1}{2}, -\frac{3}{4}, -1, -\frac{5}{4}, \ldots$



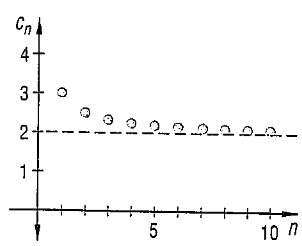

"a"

$b_n = \frac{n^2}{4}$: $\frac{1}{4}, 1, \frac{9}{4}, 4, \frac{25}{4}, \ldots$



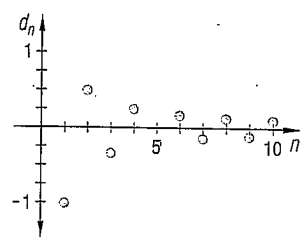

"b"

$c_n = \frac{2n+1}{n}$: $\frac{3}{1}, \frac{5}{2}, \frac{7}{3}, \frac{9}{4}, \frac{11}{5}, \ldots$




"c"

$d_n = \frac{(-1)^n}{n}$: $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \ldots$




"d"

What happens to each graphed sequence as $n$ gets very large?

"a"

"b"

"c"

"d"

Remember limit notation: as "n" (our I.V.) gets very large, the output $a_n$ (D.V.) approaches a specific quantity:

$\lim_{n \to \infty} (f(x)) =$ ____

Define convergent:

Define divergent:

Which of the four sequences are divergent?

What is the limit of ?

What is the limit of ?

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Harmonic sequence: $a_n = \frac{1}{n}$ What is $\lim_{n \to \infty} (a_n)$?

Restate:

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Theorem: $\lim_{n \to \infty} (c) = c$ assume c is a constant.

Why?

Does $a_n = \frac{2n+1}{n}$ converge?

Thm #2 $\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$

Restate:

Thm #3 $\lim_{n\to\infty} (a_n \cdot b_n) = \left(\lim_{n\to\infty} (a_n)\right)\left(\lim_{n\to\infty} (b_n)\right)$

Restate:

Find $\lim_{n\to\infty} \frac{2-n}{4-3n}$

Similar to end behavior of asymptotes of a rational function, you compare the degrees of num. and denominator (P. 221)
If...

1. degree of num. = degree of denom.

2. degree of num. < degree of denom

3. degree of num. > degree of denom.

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Determine if the sequence converges. If so, find its limit.

a) $a_n = \dfrac{3n}{n+1}$

b) $b_n = \dfrac{n^3+2}{n^2+n}$

c) $c_n = \dfrac{5n^2}{n^3+1}$