# Chapter 6, Part 1

## Vectors in the Plane

*Precalculus*

# Why Vectors?

- Some quantities, like temperature, height, area, and volume can be represented by a single real number that indicates magnitude or size
- Other quantities, like acceleration, velocity, and force have both magnitude and direction
  - Use ordered pairs to help describe magnitude and direction
  - While (a, b) represents a point in the plane, it also determines a directed line segment with its tail at the origin and its head at (a, b).
    - This is called the **position vector of (a, b)**

- The length of the arrow is its **<u>magnitude</u>**
  - magnitude = $|v|$
- The direction to which the arrow points is the vector's direction
- A vector can be notated by **v**, or <a, b>
  - <a, b> is called the component form of the vector, these are used to show a vector instead of an ordered pair

- In <a, b>, "a" is the horizontal component of the vector, and "b" is the vertical component of the vector
- You may also see a vector written in **standard form**:
  <a, b> = a**i** + b**j**
- A **zero vector** has zero length and no direction. It's component form is <0, 0>
- A vector has a tail point called the **initial point** and a head point called the **terminal point.**

- Two arrows (vectors) with the same length pointing in the same direction represent the same vector <a, b>. They are called **equivalent vectors.**

- To find the values of “a” and “b”, use the HMT (head minus tail) rule: Given initial point $(x_1, y_1)$ and terminal point $(x_2, y_2)$, the component form is found by $\langle x_1 - x_2 , y_1 - y_2 \rangle$

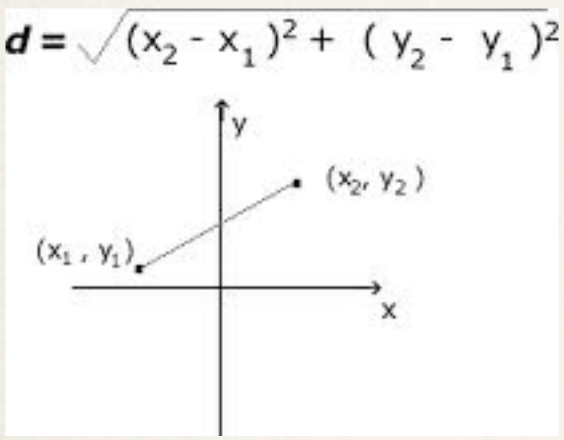
# You try...

- An arrow has an initial point (2, 3) and terminal point (7, 5). What vector does it represent?

- An arrow represents the vector <-3, 6> with an initial point (3, 5) . What is the terminal point?

# Formula for Magnitude

- Because the magnitude of a vector is the length of the arrow, the distance formula is used to determine the magnitude.




- The horizontal component of the vector is “a” and the vertical component of the vector is “b”, so the formula for magnitude is a version of Pythagorean Theorem:

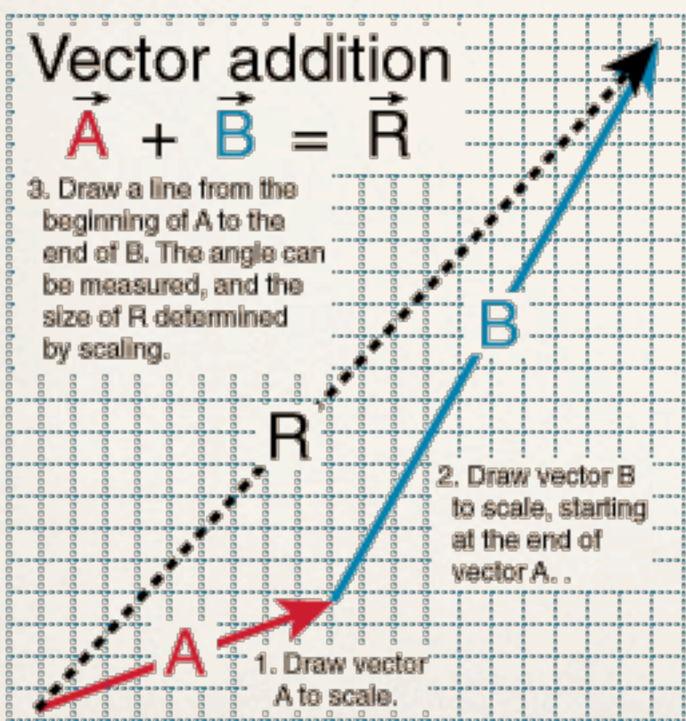
$$|\mathbf{v}| = \sqrt{(a^2 + b^2)}$$

# Vector Addition

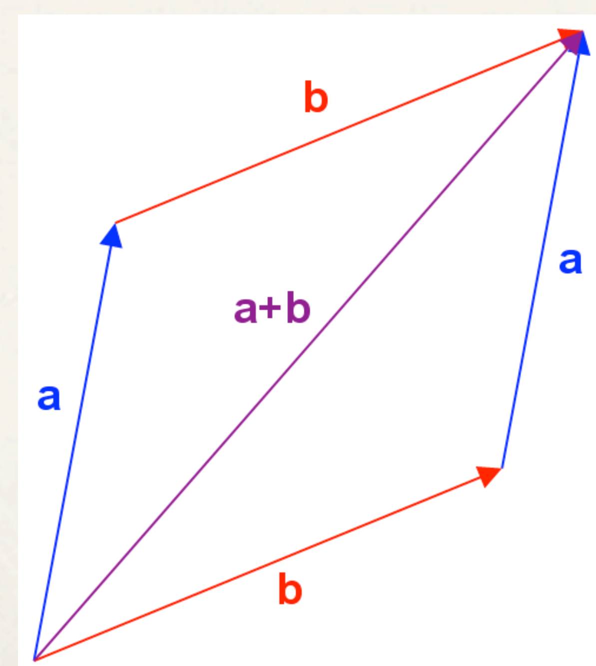
- Let **u** = $<u_1, u_2>$ and **v** = $<v_1, v_2>$. The sum (also called the **resultant**) of the vectors **u** and **v** is

$$\mathbf{u} + \mathbf{v} = <u_1+v_1,\ u_2+v_2>$$

Tail-to-head representation




Parallelogram representation



# Scalar Multiplication

- To multiply by a scalar is to use Distributive Property.
- Let **u** = $<u_1, u_2>$ and $k$ be a scalar, then $k$**u** = $<ku_1, ku_2>$

- A **<u>unit vector</u>** has a length of one unit
  - $|\mathbf{u}| = 1$
  - A unit vector is found by: $\mathbf{u} = \mathbf{v} \div |\mathbf{v}|$

# You try...

- Find the unit vector in the direction of:
  - $\mathbf{u} = <6, -2>$
  - $\mathbf{w} = 7\mathbf{i} + 7\mathbf{j}$

# Direction Angles (again...)

- From chapter 4 you should remember that direction is measured in different ways, especially in navigation (i.e. *bearing*).
- In vectors, we specify the direction of a vector **v** using its **direction angle**, the angle $\theta$ that **v** makes with the positive $x$-axis.
- Using what you learned from chapter 4, the horizontal component of **v** is $|\mathbf{v}|\cos\theta$ and the vertical component of $\mathbf{v}\theta$ is $|\mathbf{v}|\sin\theta$

$$a = |\mathbf{v}|\cos\theta \quad \text{and } b = |\mathbf{v}|\sin\theta$$

- To solve for a = $|\mathbf{v}|\cos\theta$ and b = $|\mathbf{v}|\sin\theta$ is to **resolve the vector**.

Try this...

Find the component form of question #29 on page 464

Find the magnitude and direction of the vector described by $<-1, 2>$

# Applications of Vectors

- The **velocity** of a moving object is a vector because velocity has both magnitude and direction. The magnitude of velocity is **speed**.