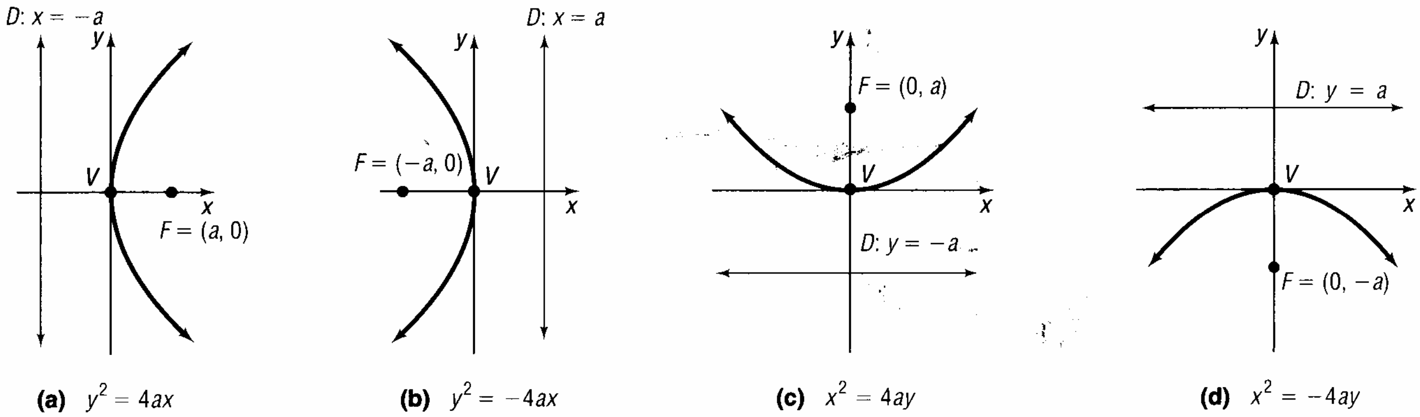
9.3 The Parabola

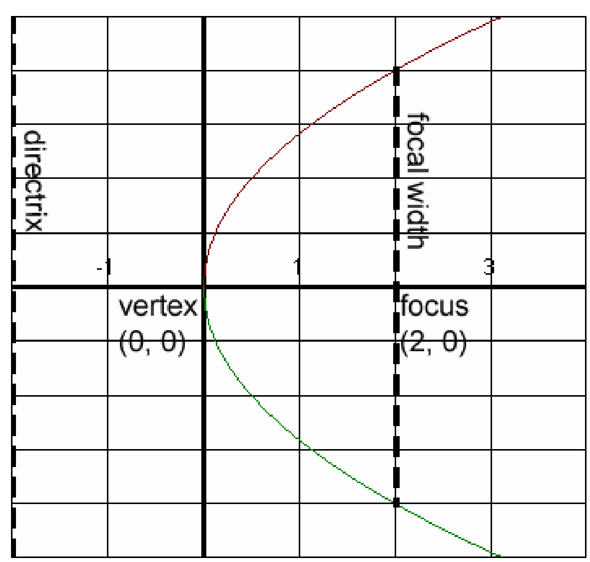
In this section we will be looking at the parabola in more detail. The pictures below show the four different ways a parabola can be drawn that are all centered at the origin. The V represents the vertex of the parabola. The F is called the **focus**. All rays that hit the parabola will be directed through the focus. That is why satellite dishes are made this way. The focus the incoming signals. The D is called the **directrix**, and this is always behind the parabola. Notice below that the parabola can have four different equations depending on which way it is orientated. Each formula has a letter ‘a’ in it. This a value is important since it will tell you how far the vertex is from the focus and how far the vertex is from the directrix.



The quantity $|4a|$ is called the **focal width**. A focal width is the length of a vertical or horizontal line that passes through the focus and touches the parabola on each end.

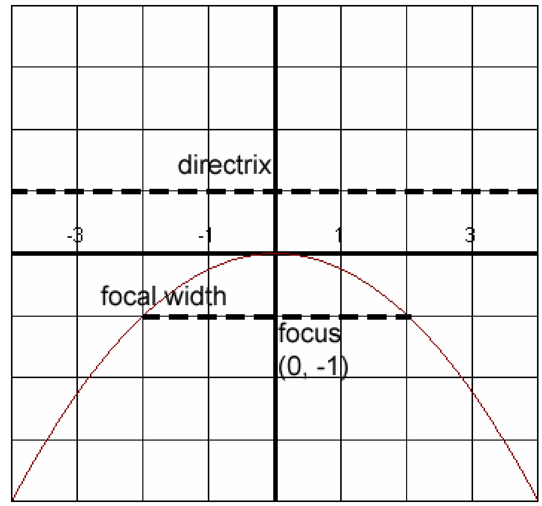
EXAMPLE: Graph $y^2 = 8x$ and identify the directrix, focus, and focal width.

First we need to find out the ‘a’ value. From the picture above we are going to use the general equation $y^2 = 4ax$. Therefore we know that $4a = 8$. Then we know that a = 2. We can find the focal width by using $|4(2)| = 8$. According to our general equation it says that the directrix is $x = -2$. We also know that the focus is at (2, 0). The vertex is at (0, 0). To draw the graph we first plot the vertex, focus, and draw the directrix. Then we will put in the focal width. Since the focal width is 8 this means from our vertex we will go up 4 and down 4 for a total distance of 8. This will give us two points on the graph so it will give us an aid in sketching. We know the graph opens to the right because we are using model (a) from the above pictures since it matches our equation.

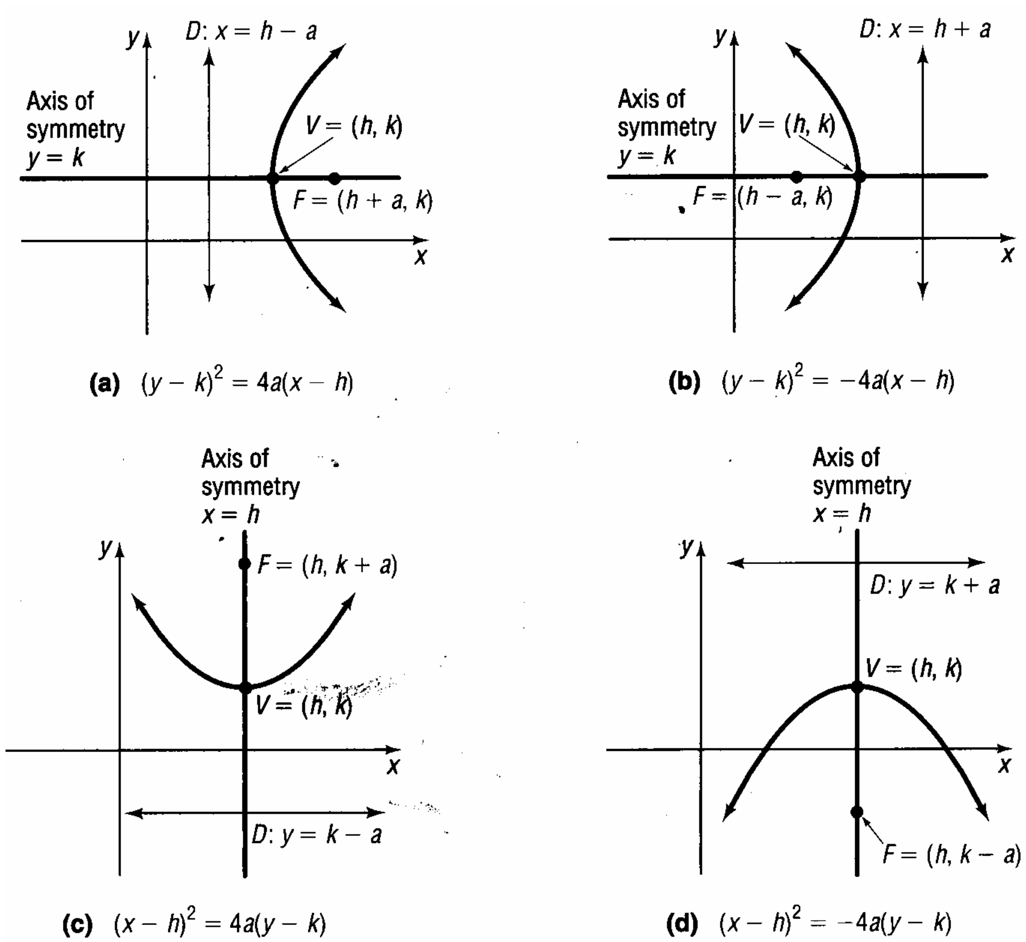


EXAMPLE: Graph $x^2 = -4y$ and identify the directrix, focus, and focal width.

First we need to find out the 'a' value. From the picture above we are going to use the general equation $x^2 = -4ay$. Therefore we know that $-4a = -4$. Then we know that a = 1. We can find the focal width by using $|-4(1)| = 4$. According to our general equation it says that the directrix is $y = 1$. We also know that the focus is at (0, -1). The vertex is at (0, 0). To draw the graph we first plot the vertex, focus, and draw the directrix. Since we are looking at graph model (d) above we know that our graph must open down. For the focal width we will go to the focus and go to the left and to the right a distance of 2 so that our total focal width is 4 as we found earlier.

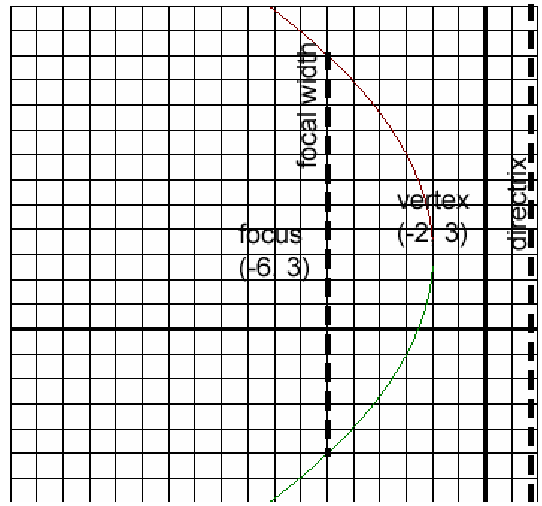


Equations for the Parabola centered at (h, k).



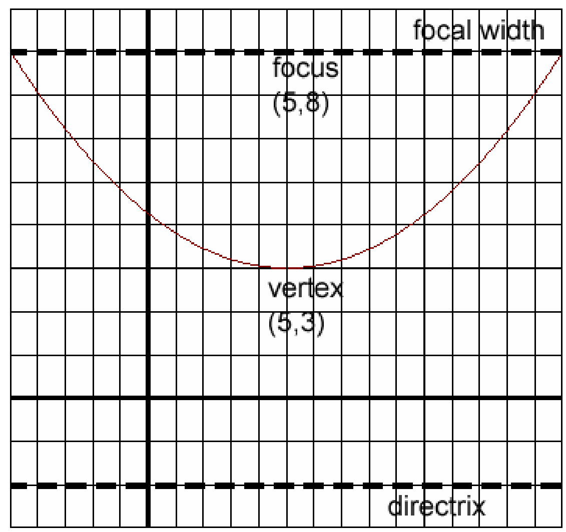
EXAMPLE: Graph $(y-3)^2 = -16(x+2)$ and identify the directrix, focus, and focal width.

First we need to find out the 'a' value. From the picture above we are going to use the general equation $(y-k)^2 = -4a(x-h)$. We know that $-4a = -16$. Then we know that a = 4. We can find the focal width by using $|-4(4)| = 16$. We can also find the vertex. It is the opposite sign of what appears in the equation. So our vertex is (-2, 3). We want to first plot the vertex. From the model we are using we know that the parabola opens to the left. That means the focus is to the left. From the vertex we will go 4 places to the left since a is 4. So we know our focus is at (-6, 3). From the vertex we can go 4 places to the right and this will be our directrix. The equation of the directix will be x = 2. Through our focus we will go up 8 and down 8 since the focal width is 16. This will give us two more points on the graph so this will help us draw the sketch.



EXAMPLE: Graph $(x-5)^2 = 20(y-3)$ and identify the directrix, focus, and focal width.

First we need to find out the 'a' value. From the picture above we are going to use the general equation $(x-h)^2 = 4a(y-k)$. We know that $4a = 20$. Then we know that a = 5. We can find the focal width by using $|-4(5)| = 20$. We can also find the vertex. It is the opposite sign of what appears in the equation. So our vertex is (5, 3). We want to first plot the vertex. From the model we are using we know that the parabola opens up. That means the focus is above the vertex. From the vertex we will go up 5 places since a is 5. So we know our focus is at (5, 8). From the vertex we can go down 5 and this will be our directrix. The equation of the directrix is y = -2. Through our focus we will go to the left 10 and to the right 10 since the focal width is 20. This will give us two more points on the graph so this will help us draw the sketch.

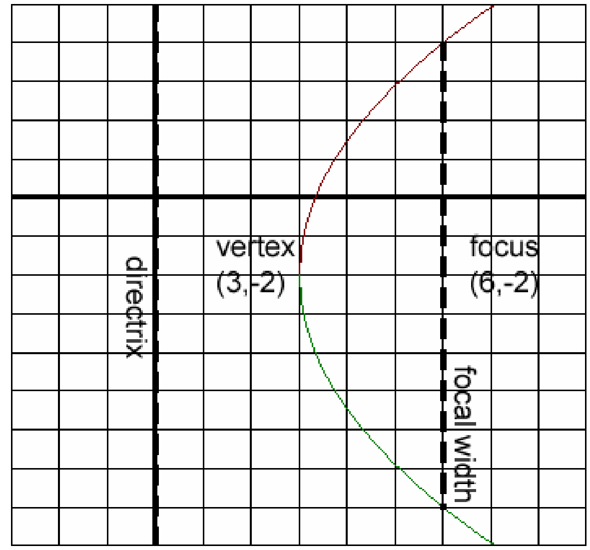


EXAMPLE: Graph $y^2 + 4y - 12x + 40 = 0$ and identify the directrix, focus, and focal width.

This is not in the proper form for us to graph, so we need to do something to this equation first. I will leave the y terms on the right hand side. Then I will move the other terms to the right side. Then I will have: $y^2 + 4y = 12x - 40$. You need to complete the square on the left hand side. We take the number 4 and divide it by 2 and then square it. You will get 4. We want to add this to both sides of the equation: $y^2 + 4y + 4 = 12x - 40 + 4$. Now we can simplify the right hand side and factor the left side: $(y+2)^2 = 12x - 36$. Finally we can factor the right side: $(y+2)^2 = 12(x-3)$

Now we have it in the proper form, so we can now graph it on the next page…

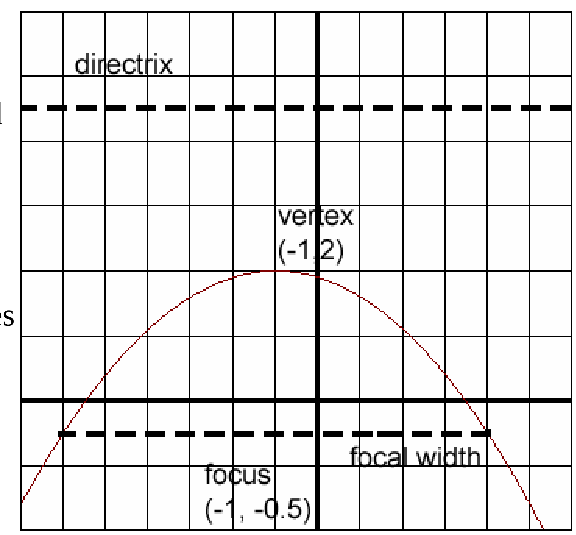
First we need to find out the 'a' value. From the picture above we are going to use the general equation $(y-k)^2 = 4a(x-h)$. We know that $4a = 12$. Then we know that a = 3. We can find the focal width by using $|-4(3)| = 12$. We can also find the vertex. It is the opposite sign of what appears in the equation. So our vertex is (3, -2). We want to first plot the vertex. From the model we are using we know that the parabola opens to the right. That means the focus is to the right of the vertex. From the vertex we will go 3 places to the right. So we know our focus is at (6, -2). From the vertex we can go 3 places to the left and this will be our directrix. The equation of the directrix is x = 0. Through our focus we will go up 6 and down 6 since the focal width is 12. This will give us two more points on the graph so this will help us draw the sketch.



EXAMPLE: Graph $2x^2 + 4x + 20y - 38 = 0$ and identify the directrix, focus, and focal width.

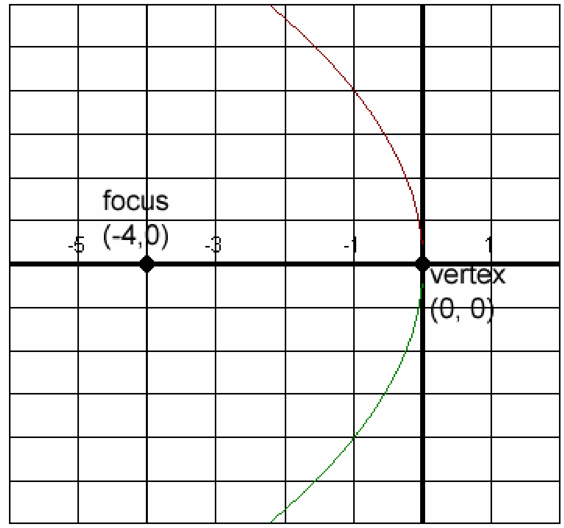
This is not in the proper form for us to graph, so we need to do something to this equation first. First we need to divide the entire equation by 2: $x^2 + 2x + 10y - 19 = 0$. I will leave the x terms on the right hand side. Then I will move the other terms to the right side. Then I will have: $x^2 + 2x = -10y + 19$. You need to complete the square on the left hand side. We take the number 2 and divide it by 2 and then square it. You will get 1. We want to add this to both sides of the equation: $x^2 + 2x + 1 = -10y + 19 + 1$. Now we can simplify the right hand side and factor the left side: $(x+1)^2 = 10y + 20$. Finally we can factor the right side: $(x+1)^2 = -10(y-2)$

First we need to find out the 'a' value. From the picture above we are going to use the general equation $(x-h)^2 = -4a(y-k)$. We know that $-4a = -10$. Then we know that a = 2.5. We can find the focal width by using $|-4(2.5)| = 10$. We can also find the vertex. It is the opposite sign of what appears in the equation. So our vertex is (-1, 2). We want to first plot the vertex. From the model we are using we know that the parabola opens down. That means the focus is below the vertex. From the vertex we will go down 2.5 places So we know our focus is at (-1, -0.5). From the vertex we can go up 2.5 places and this will be our directrix. The equation of the directrix is y = 4.5. Through our focus we will go to the left 5 and to the right 5 since the focal width is 10. This will give us two more points on the graph so this will help us draw the sketch.



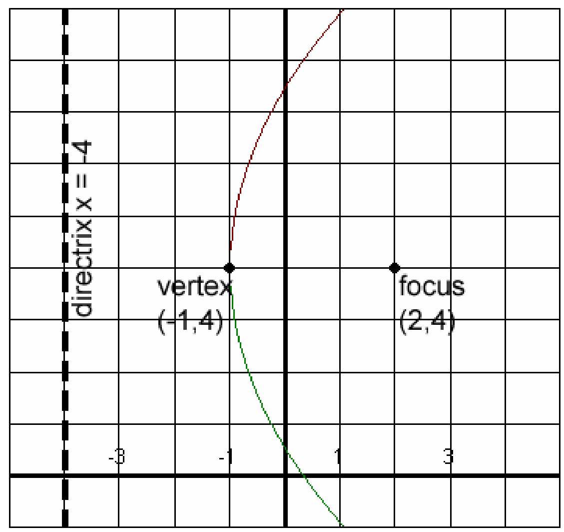
EXAMPLE: Find the equation of the parabola if it is known the focus is (-4, 0) and the vertex is (0, 0).

For this one we know that the focus is 4 units away from the vertex. This means that a is 4. Since it is to the left of the vertex (see graph) we know the graph is opening up to the left. A parabola opening up to the left has the equation $y^2 = -4ax$. If we put in a 4 for a we will get our solution: $y^2 = -16x$.



EXAMPLE: Find the equation of the parabola if it is known the focus is (2, 4) and the directrix is at x = -4.

We can first plot (2, 4) and then draw in the directrix. The vertex has to be exactly half way between the focus and directrix. Then we know the vertex is at (-1, 4). The distance from the vertex to either the focus or the directrix is 3, so we know that a is 3. The graph will open towards the right since the focus is to the right of the vertex. Then we know the general equation is: $(y-k)^2 = 4a(x-h)$. We know that a is 3, h is -1 and k is 4. We plug these into the formula to get: $(y-4)^2 = 12(x+1)$.



EXAMPLE: Find the equation of the parabola if it is known the vertex is (3, 0) and the directrix is at y = 2.

We first plot (3, 0) and draw y = 2. The distance from the vertex to the directrix is 2. This tells us that a = 2. Since the directrix is horizontal and the vertex is below this line we know the graph opens down, so the general equation is: $(x-h)^2 = -4a(y-k)$. We can now plug in 2 for a, 3 for h, and 0 for y. Our equation will be: $(x-3)^2 = -8y$.