

Solving linear systems using the substitution method

Solving Systems
of Linear
Equations

Another method used to solve linear systems is called the substitution method. Follow these four steps when using this method.

1. Solve one of the equations for one of its variables.
2. Substitute the expression found in step 1 into the other equation and solve for the other variable.
3. Substitute the value found in step 2 into the expression found in step 1 and solve.
4. Check the solution in each of the original equations.

Solve

$$x - y = 2$$

$$2x + 3y = -6$$

$$1. x = y + 2$$

$$2. 2(y + 2) + 3y = -6$$

$$2y + 4 + 3y = -6$$

$$5y = -10$$

$$3. y = -2$$

$$\text{So, } x = 0.$$

Solve for x .

Substitute $y + 2$ for x .

Solve for y .

Substitute y -value into $x = y + 2$.

$$\text{Check } 0 - (-2) = 2 \quad 2(0) + 3(-2) = -6$$

$$2 = 2$$

$$-6 = -6$$

true

true

Thus, the solution of this system is $(0, -2)$.

Solve one of the equations for one of its variables.

$$1. 3x + y = 4$$

$$-2x - 5y = 7$$

$$2. x - 5y = 10$$

$$4x + 2y = 9$$

$$3. -7x + 3y = 14$$

$$6x + y = -3$$

$$4. 8x - 5y = -10$$

$$-x - 5y = 11$$

Solve each system using the substitution method.

$$5. y = x$$

$$4x - 3y = -1$$

$$6. 5x - y = -5$$

$$x - y = 3$$

$$7. 2x + y = -11$$

$$x + 2y = -1$$

$$8. x = 3y + 10$$

$$-2x - 5y = 2$$

$$9. 3y - 4 + x = 0$$

$$5x + 6y = 11$$

$$10. 8x + 5y = 7$$

$$-x - y = 1$$

$$11. y = x + 3$$

$$x + 2y = -3$$

$$12. x + 2y = 2$$

$$7x + 10y = 2$$

$$13. 2x - y = 5$$

$$x - 3y = 0$$

Solving linear systems using multiplication and addition

Solving Systems
of Linear
Equations

To solve a linear system using both multiplication and addition, use the following steps:

1. Arrange the equations with like terms in columns.
2. Look at the coefficients of x and y in both equations. Multiply one or both equations by a specific number that gives new coefficients for x (or y) that are opposites.
3. Add the equations and solve for the variable that remains.
4. Substitute the value found in step 3 into one of the original equations and solve for the other variable.
5. Check the solution in both of the original equations.

$\begin{array}{r} \text{Solve } 3x - y = 2 \\ -2x + 4y = 2 \end{array}$	$\begin{array}{r} 4(3x - y) = 4(2) \\ 12x - 4y = 8 \\ + -2x + 4y = 2 \\ \hline 10x = 10 \\ x = 1 \end{array}$	<p>Multiply the first equation by 4. Add. Solve for x, the remaining variable.</p>
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Substitute the x -value into one of original equations and solve for y . $3(1) - y = 2$

$\begin{array}{r} \text{Check } 3(1) - 1 = 2 \\ 3 - 1 = 2 \\ 2 = 2 \text{ True} \end{array}$	$\begin{array}{r} -2(1) + 4(1) = 2 \\ -2 + 4 = 2 \\ 2 = 2 \text{ True} \end{array}$	$\begin{array}{r} 3(1) - y = 2 \\ 3 - y = 2 \\ y = 1 \end{array}$
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Thus, the solution of the linear system is $(1, 1)$.

State the best number to be multiplied by one equation to eliminate one of the variables using addition.

1. $x - 2y = 4$
 $-3x + 5y = -3$

2. $-3x - 6y = 11$
 $2x + y = 4$

3. $7x + 2y = 5$
 $-x - 6y = 8$

4. $6x - 5y = 8$
 $-2x - y = 3$

Solve each system using multiplication and addition.

5. $4x + y = 5$
 $2x + 5y = 7$

6. $7x + 2y = -4$
 $3x + 6y = 24$

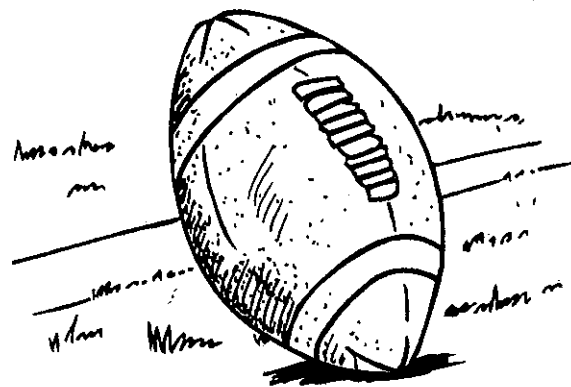
7. $-2x + 5y = 8$
 $-5x + 3y = 1$

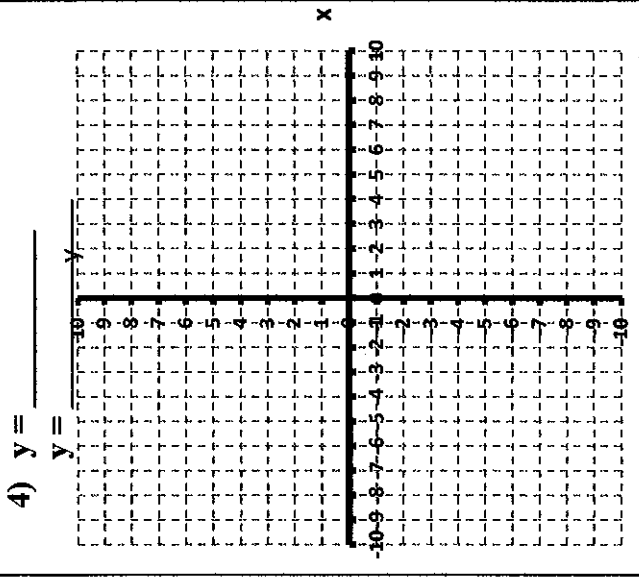
8. $-5x + 2y = 22$
 $-3x + 2y = 6$

9. $x + y = 0$
 $2x - 11y = 39$

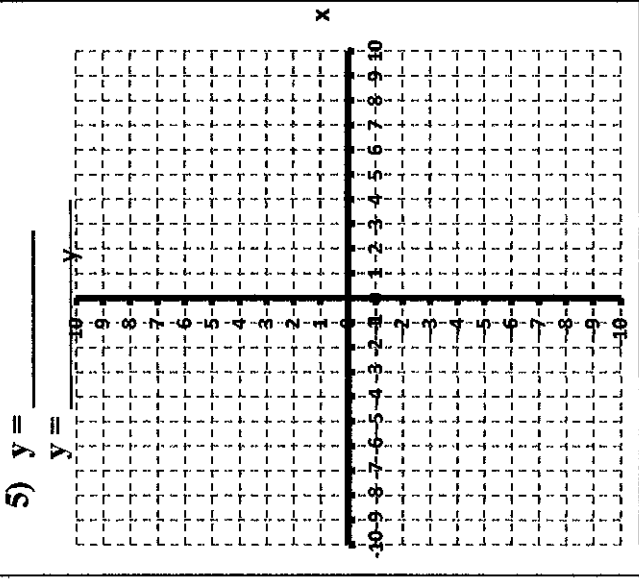
10. $-3x + 4y = 1$
 $x + 2y = 3$

11. $2x + 3y = 12$
 $5x + 2y = 8$

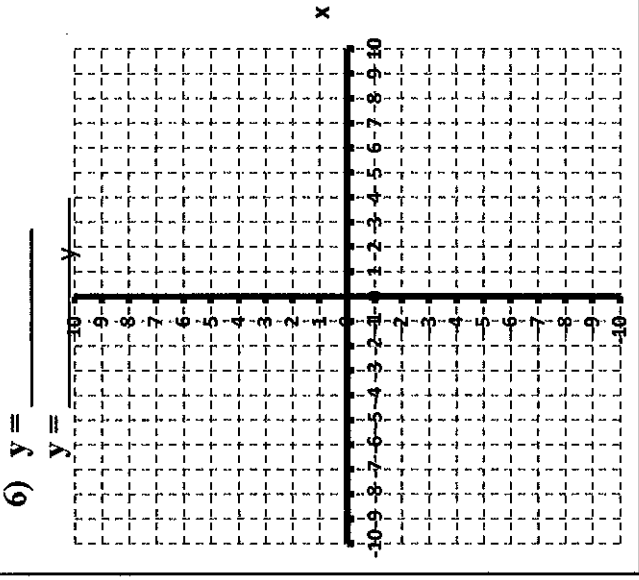




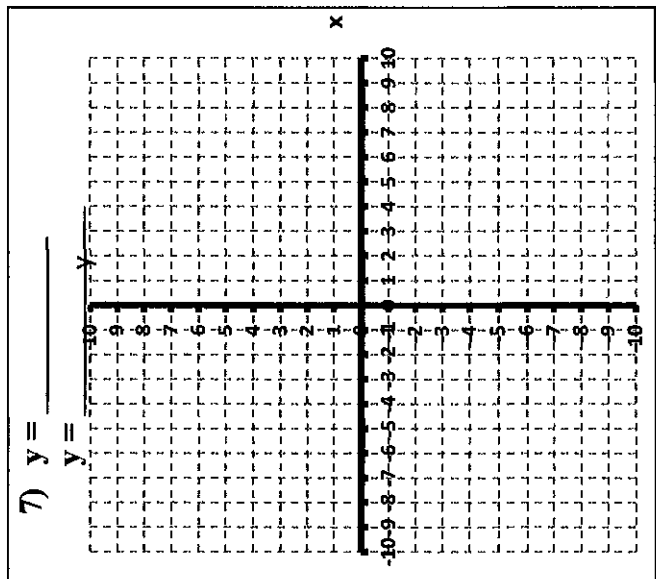
Solution: _____



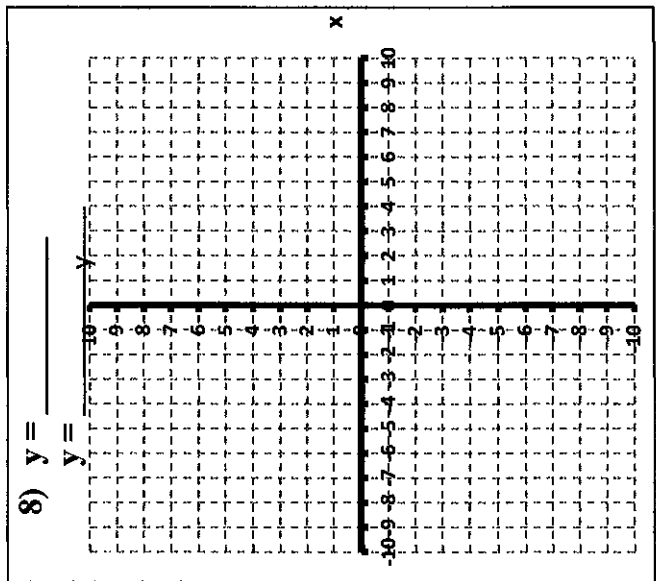
Solution: _____



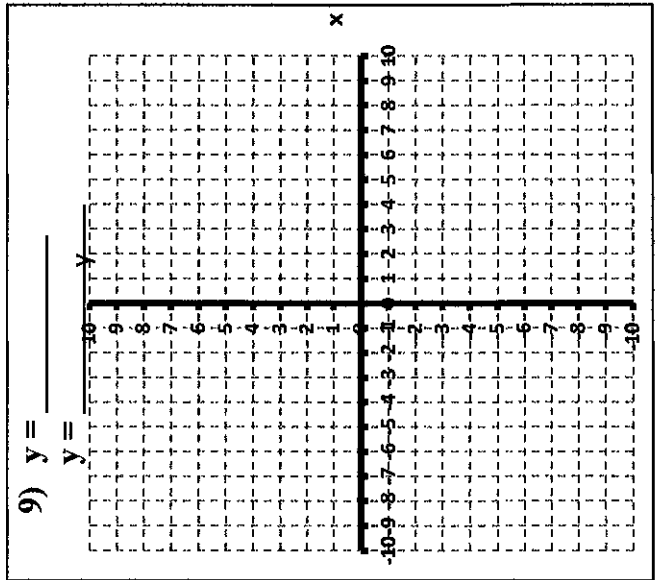
Solution: _____



Solution: _____

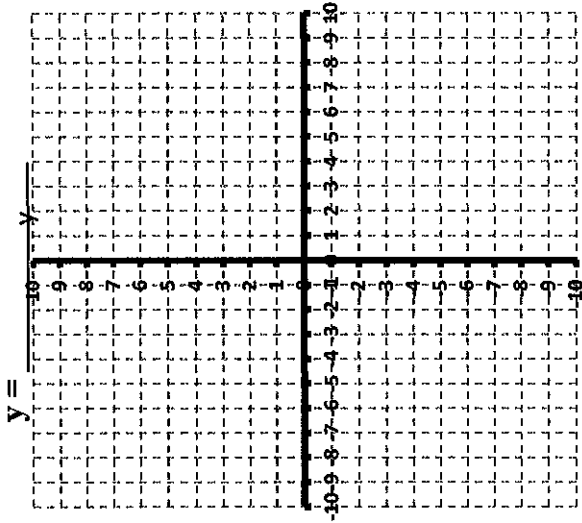


Solution: _____



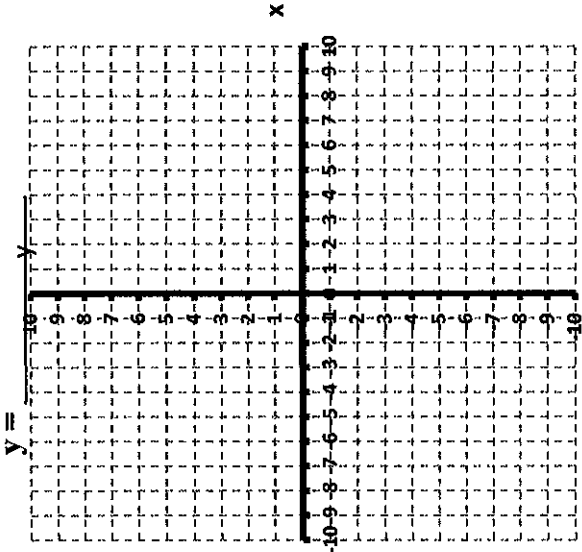
Solution: _____

10) $y =$ _____



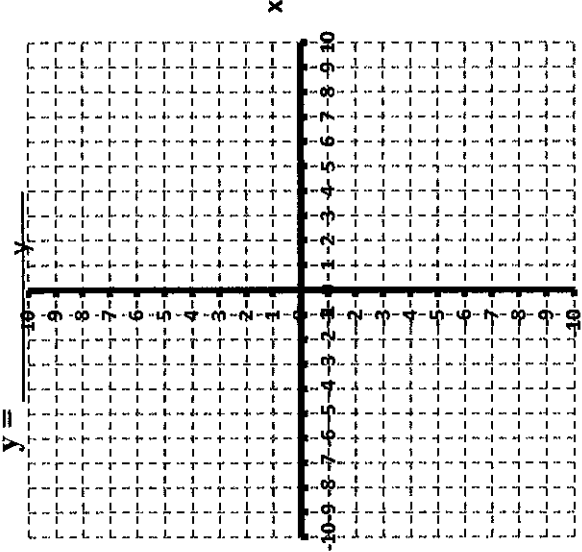
Solution: _____

11) $y =$ _____



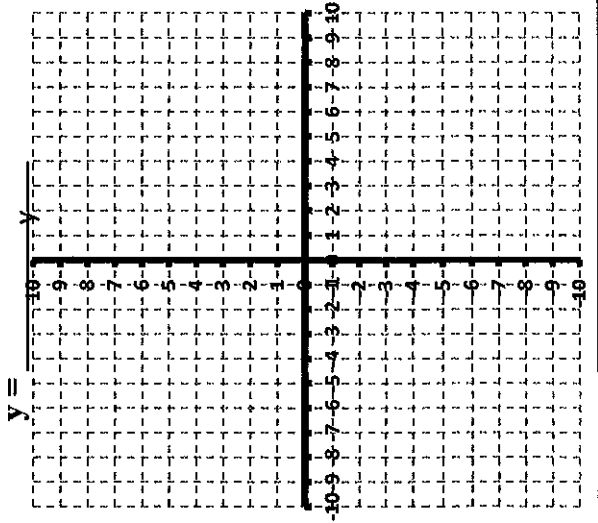
Solution: _____

12) $y =$ _____



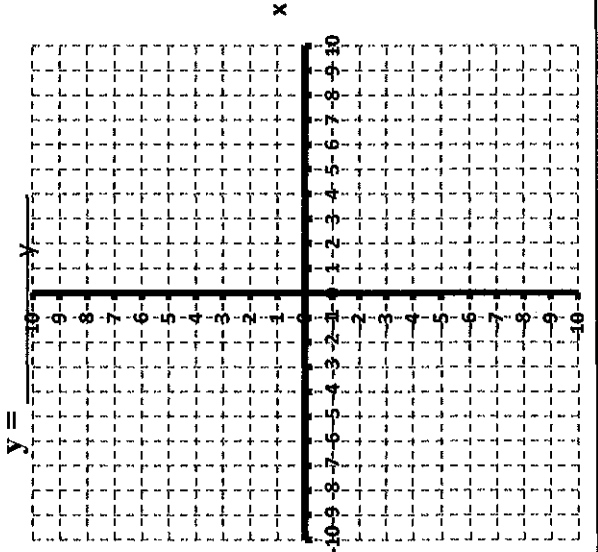
Solution: _____

5) $y =$ _____



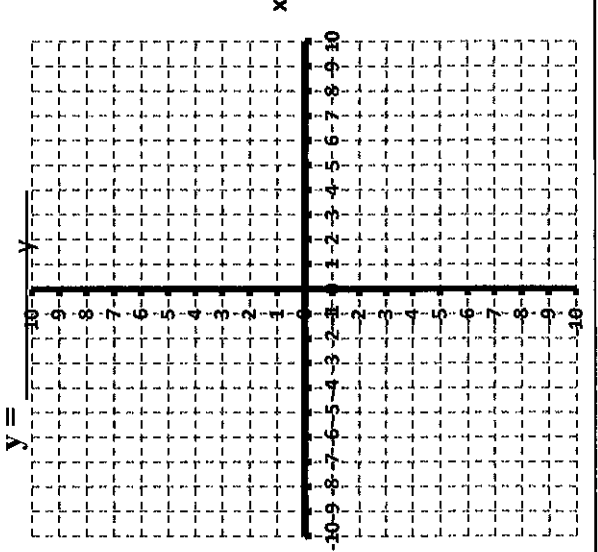
Solution: _____

6) $y =$ _____



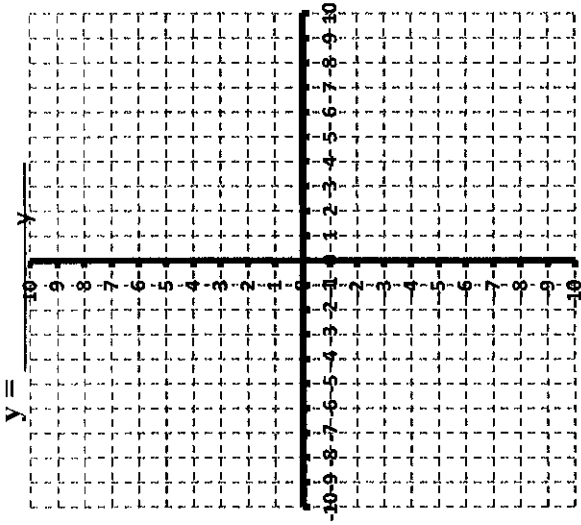
Solution: _____

7) $y =$ _____



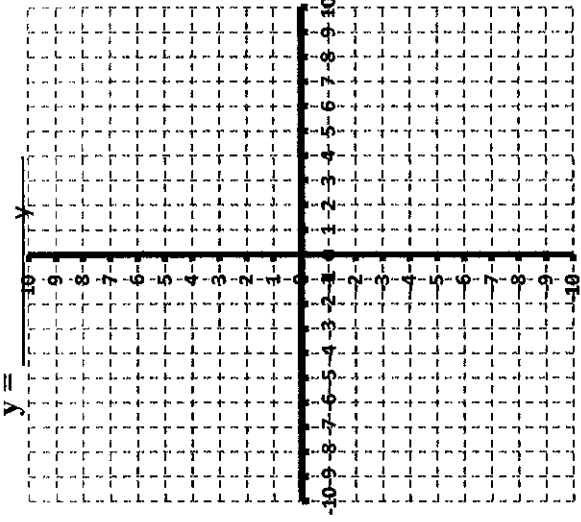
Solution: _____

8) $y =$ _____



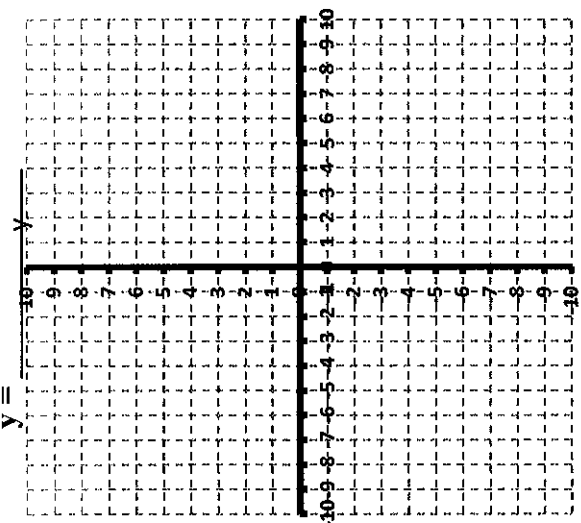
Solution: _____

9) $y =$ _____



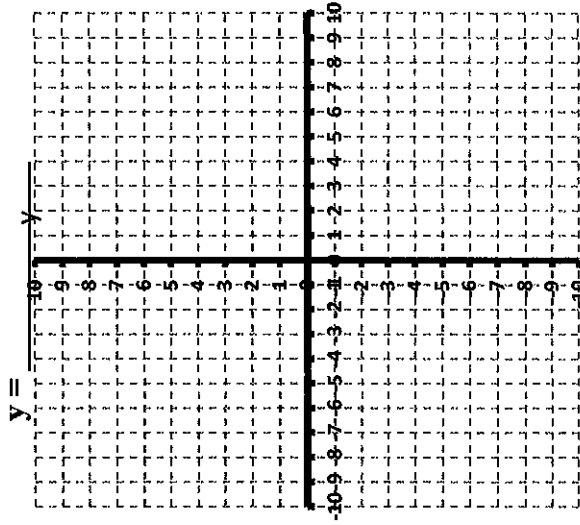
Solution: _____

10) $y =$ _____



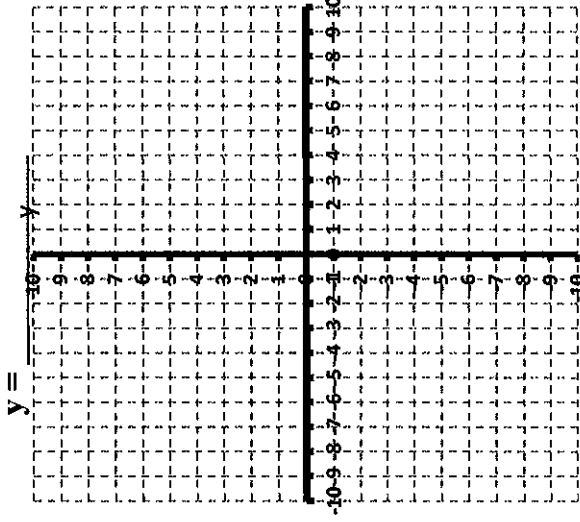
Solution: _____

11) $y =$ _____



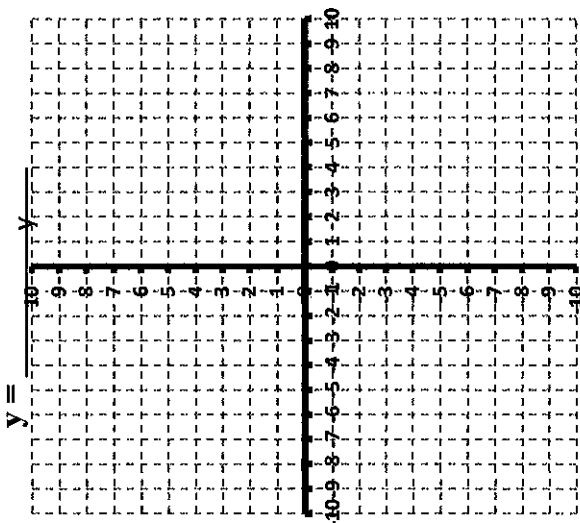
Solution: _____

12) $y =$ _____



Solution: _____

13) $y =$ _____



Solution: _____

Graphing linear inequalities with two variables

Working with Inequalities
and Absolute Values

A linear inequality can be written in the following forms:

$$ax + by < c$$

$$ax + by \leq c$$

$$ax + by > c$$

$$ax + by \geq c$$

where (x, y) is an ordered pair that is a solution of the linear inequality, making the inequality true.

Is $(-2, 5)$ a solution of $2x - 6y > 12$?

$$2(-2) - 6(5) > 12$$

Substitute the x and y values into the inequality.

$$-4 - 30 > 12$$

$$-34 > 12$$

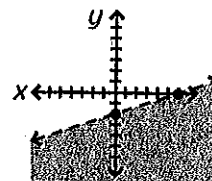
Not true. Therefore, $(-2, 5)$ is not a solution.

To sketch a graph of a linear inequality, follow these simple steps.

1. Sketch the graph of the corresponding linear equation using a dashed line for $<$ or $>$ and a solid line for \leq or \geq . Thus, separating the coordinate plane into two half planes.
2. Pick a point in each of the half planes and test each to find which one is a solution to the linear inequality.
3. Shade the half of the plane that contains the point that is a solution to the linear inequality.

The graph of the above example of $2x - 6y > 12$ is shown.

Note: Since $(-2, 5)$ was not a solution, choose another point on the other half of the plane to show what half needs to be shaded.



Write **yes** or **no** to state whether the given point is a solution of the inequality.

1. $2x - 3y \leq 7$; $(5, -4)$

2. $-x - y > 5$; $(-2, -4)$

3. $5x + 4y \geq 8$; $(-2, 6)$

4. $-7x + 8y < 12$; $(-3, 2)$

Sketch the graph of each inequality.

5. $x + y < 5$

6. $x > -1$

7. $2x - y \geq 2$

8. $y \leq 3$

9. $-3x + 4y \leq 12$

10. $5x - 2y > -10$

Solving systems of linear inequalities

Solving Systems of Linear Equations

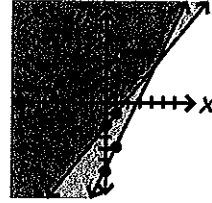
A system of linear inequalities consists of two or more inequalities with two or more variables. The solution of an inequality system is an ordered pair that is a solution of each inequality in the system. The graph of a system shows all solutions of the system.

Solve $y \geq x - 2$ by graphing.

$$2x - y \leq 6$$

Graph both lines.

Shade each line's half plane.



Thus, the solution of the linear inequality is the double-shaded region, containing all the points that are solutions of the system.

State whether each inequality would be a **solid** or a **dashed** line. Tell whether you would shade **above** or **below** the line.

1. $3x - y \leq 7$

2. $6x - y < 10$

3. $y \leq 2x - 8$

4. $4x + 5y > 2$

5. $x + y \geq -5$

6. $y < 9x + 1$

Solve each linear inequality system by graphing.

7. $x \geq 2$

8. $y \geq 3x + 1$

9. $4x - 5y < -20$

10. $x \geq -3$

$y \leq 4$

$y \leq -x - 4$

$3x + 2y \geq 6$

$y \geq 4$

$x < 2$

11. $y \leq -3$

12. $y < 5x + 2$

13. $y - x < 0$

14. $y \leq 0$

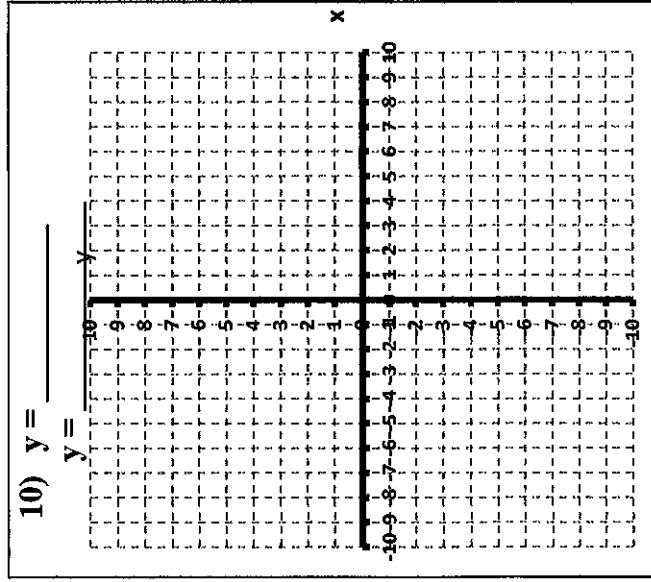
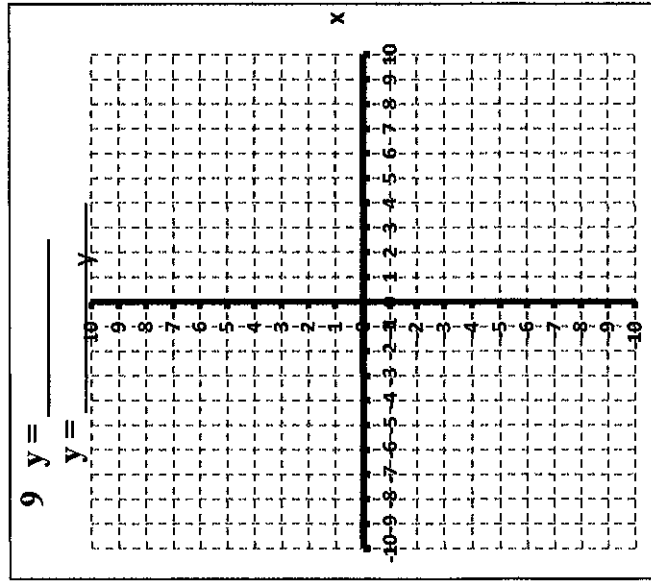
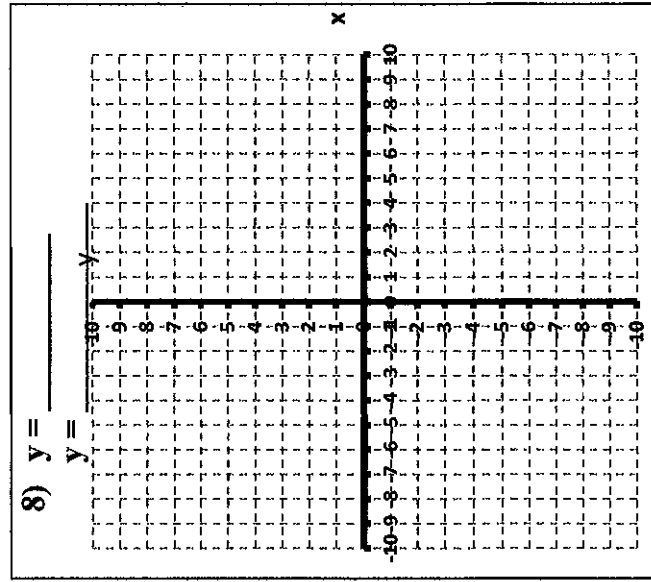
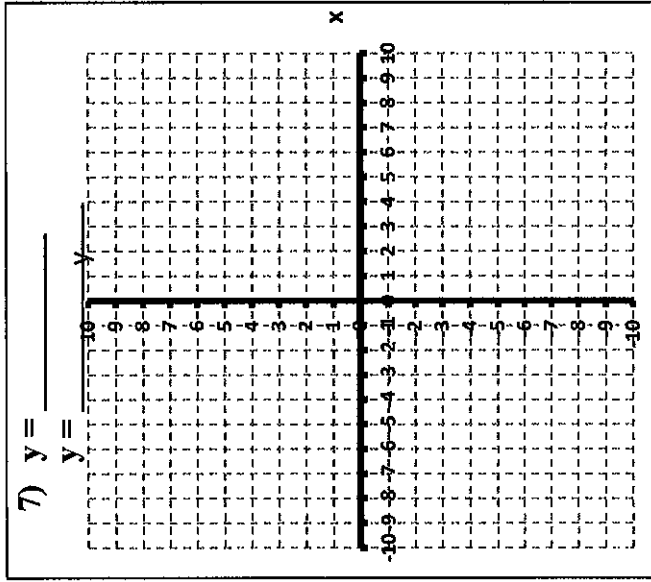
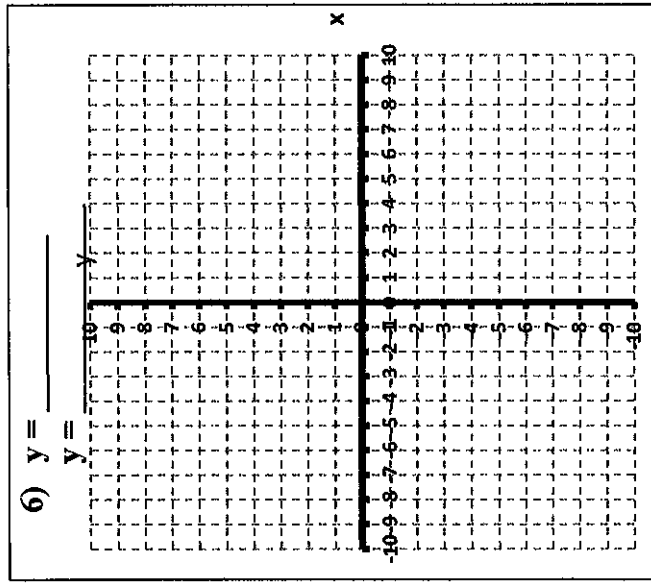
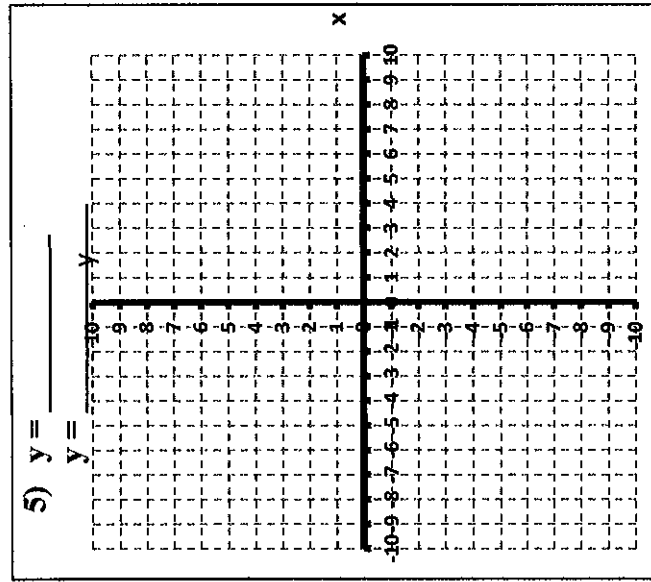
$y > 5$

$y > -2x - 3$

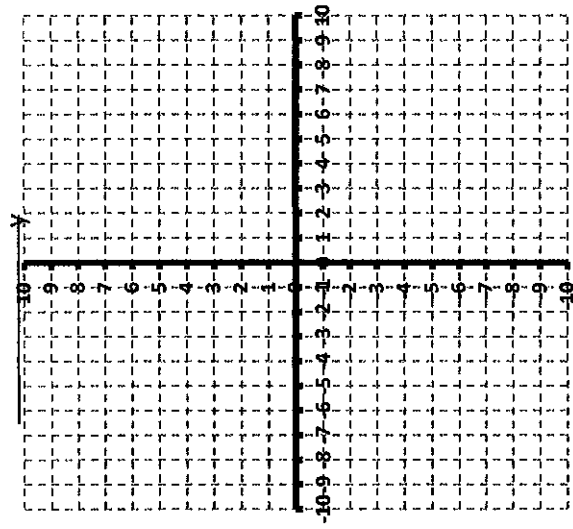
$2x - y > -3$

$x + y \leq 5$

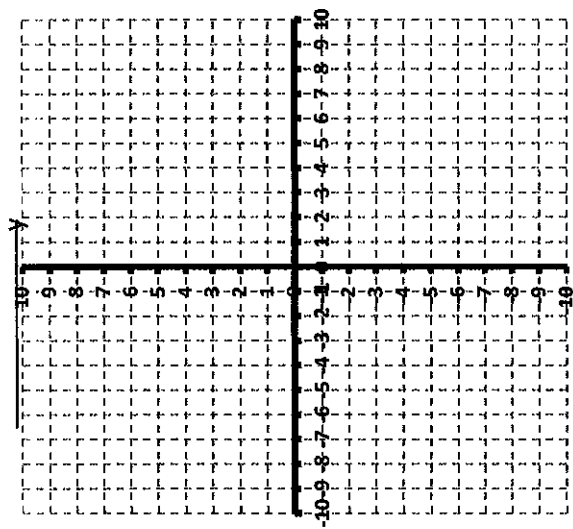
$y \geq -x + 2$



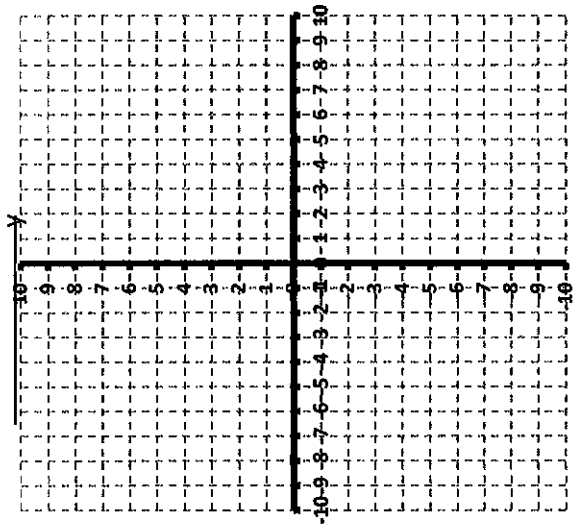
7) _____



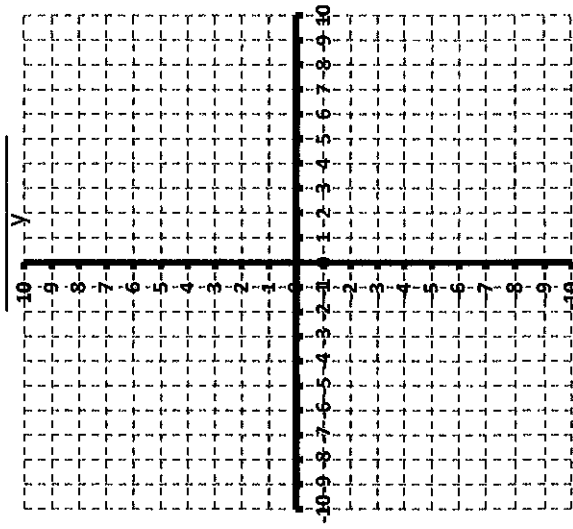
8) _____



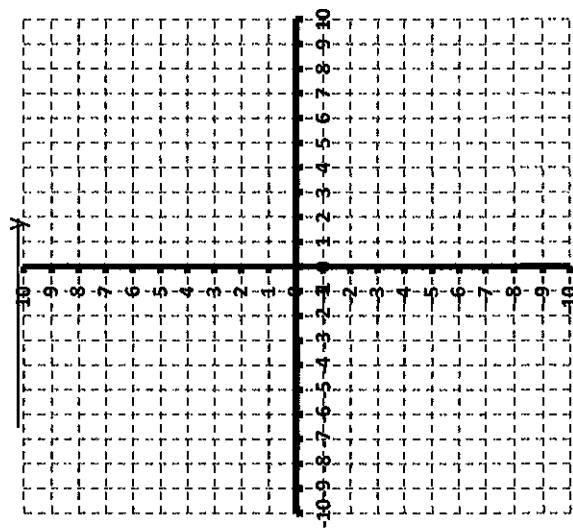
9) _____



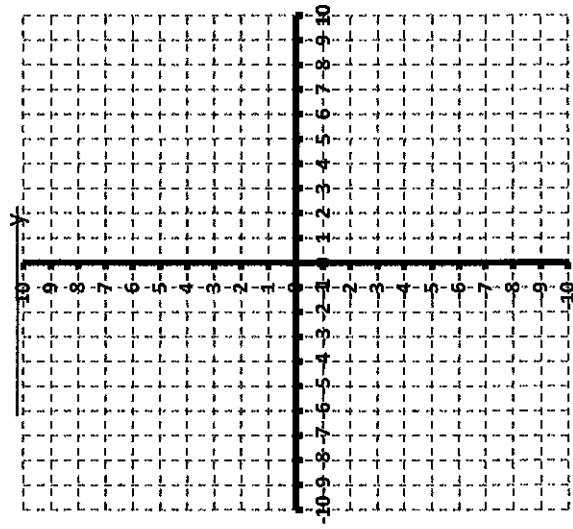
10) _____



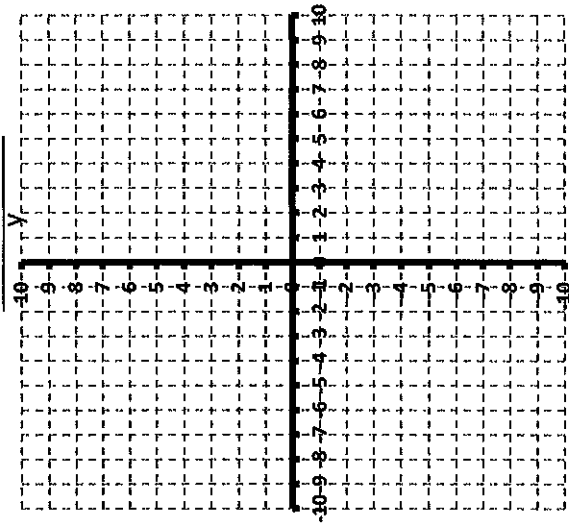
11) _____



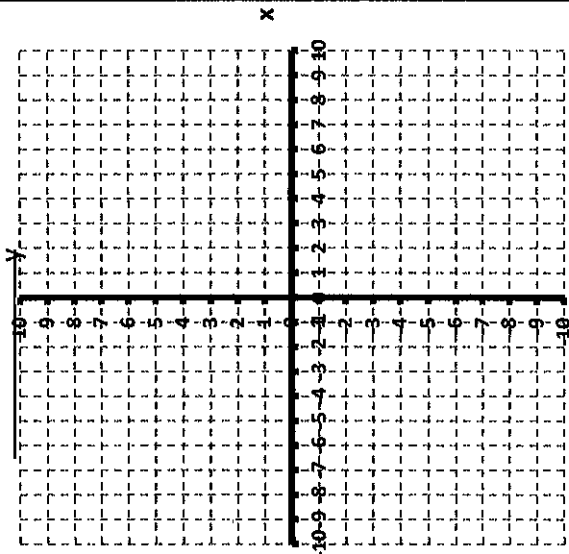
12) _____



13)

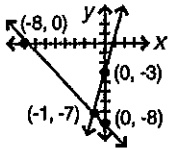


14)

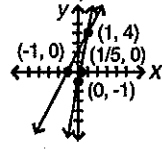


Answer Key

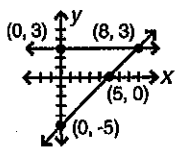
6. (-1, -7)



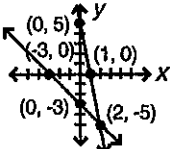
7. (1, 4)



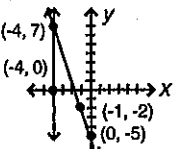
8. (8, 3)



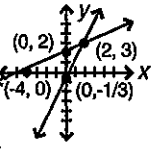
9. (2, -5)



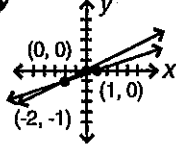
10. (-4, 7)



11. (2, 3)



12. (-2, -1)



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- 1.-4. Answers will vary. Possible answers:
 1. $y = -3x + 4$; 2. $x = 5y + 10$; 3. $y = -6x - 3$;
 4. $x = -5y - 11$; 5. (-1, -1); 6. (-2, -5); 7. (-7, 3);
 8. (4, -2); 9. (1, 1); 10. (4, -5); 11. (-3, 0);
 12. (-4, 3); 13. (3, 1)

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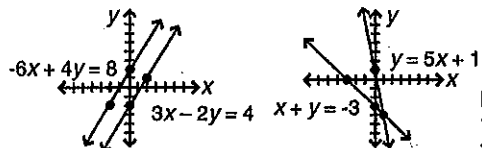
1. (3, 1/2); 2. (1, 1); 3. (-18/5, -10); 4. (9/2, 2);
 5. (-7/3, 2); 6. (-2, 3); 7. (2, -5); 8. (-1, 4);
 9. (1, -1/7); 10. (6, 2); 11. (-2, 3); 12. (-2/3, 8);
 13. Neither set of variables have opposite coefficients. To fix this, multiply the first equation by -3. Explanations will vary.

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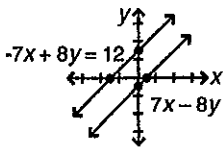
1. Multiply first equation by 3.; 2. Multiply second equation by 6.; 3. Multiply second equation by 7.; 4. Multiply second equation by -5.; 5. (1, 1); 6. (-2, 5); 7. (1, 2); 8. (-8, -9); 9. (3, -3); 10. (1, 1); 11. (0, 4)

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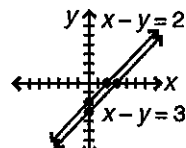
1. a, one solution; 2. c, one solution; 3. d, infinitely many solutions (same line); 4. b, no solution (parallel lines); 5. no solution 6. (1, -4), one solution



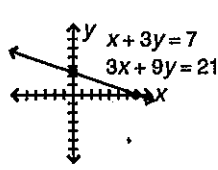
7. no solution



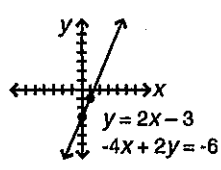
8. no solution



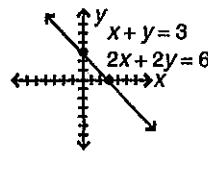
9. infinitely many solutions



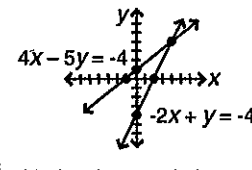
11. infinitely many solutions



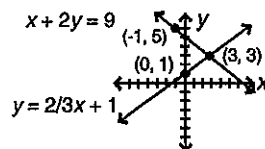
10. infinitely many solutions



12. (4, 4) one solution

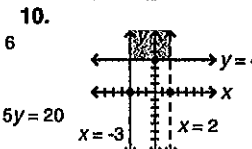
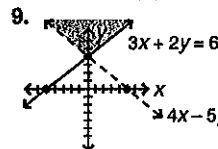
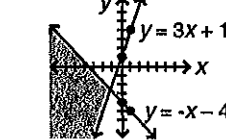
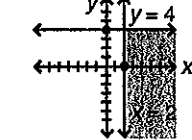


13. (3, 3) one solution

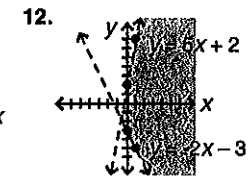
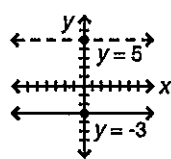


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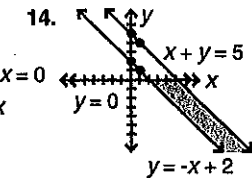
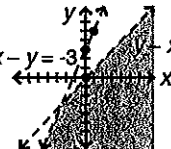
1. solid, above; 2. dashed, above; 3. solid, below; 4. dashed, above; 5. solid, above; 6. dashed, below;



11. no solution



13.

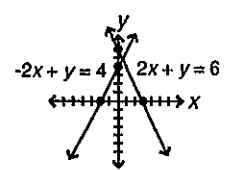
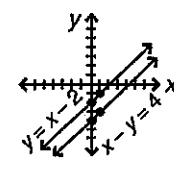


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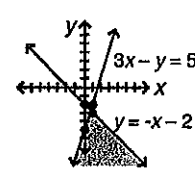
1. Josh = 2 years, Cheryl = 8 years; 2. 28, 22;
 3. Joe's = \$7.25, Jim's = \$8.75; 4. fries = \$.75, hamburger = \$1.00; 5. quarters = 16, nickels = 24

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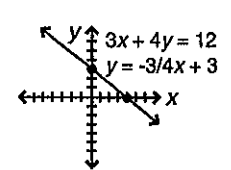
1. (-2, 1) is circled.; 2. (-5, -5) is circled.; 3. no solution 4. (1/2, 5), one solution



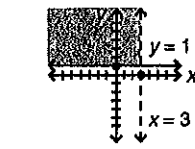
5. infinitely many solutions



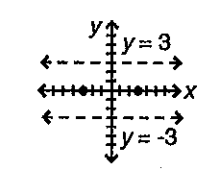
6. infinitely many solutions



7. infinitely many solutions



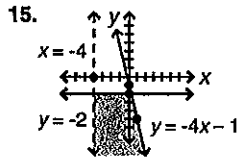
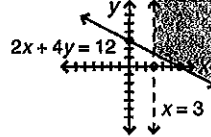
8. no solutions



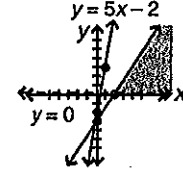
9. (2, 5); 10. (1, -1); 11. (2, 0); 12. (-4, -1); 13. mowing lawns = \$5 per hour, walking dogs: \$3 per hour

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1. yes; 2.-7. Explanations will vary.
 2. substitution, (-1, -2); 3. adding, (8, 4);
 4. substitution, (-4, -4); 5. graphing, (3, -7);
 6. substitution, (0, 1); 7. multiplication and adding, (2, 4); 8. no solution; 9. one solution, (0, 2); 10. one solution, (-1, 3); 11. infinitely many solutions; 12. infinitely many solutions; 13. no solutions;



16.



17. John's age = 12 years, Mike's age = 36 years

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1. Add the exponents.; 2. x: base, 3: exponent, x^3 : power; 3. Find the power of each factor and multiply.; 4. $3x^8$; 5. $36x^8y^6$; 6. $-96x^3y^4z^{13}$;
 7. $-4x^7$; 8. $-128x^{11}y^{10}$; 9. $-90x^{11}y^8$; 10. x^4 ;
 11. $-x^3y^8$; 12. $15x^8$; 13. $64x^{14}y^{14}z^{12}$

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1. 1; 2. $1/x^8$; 3. $x^2/3$; 4. $1/x^3$; 5. $1/x^{10}$; 6. $x^4/5$;
 7. $6x^2$; 8. $6/x^3$; 9. y^4/x^3 ; 10. $8x^3$; 11. $4/x^5$;
 12. $4/x^4y^2$; 13. $9/x^4$; 14. $1/16$; 15. 1; 16. 1;
 17. -27; 18. $1/1296$; 19. -8