

Histograms and **bar graphs** are visual ways to represent data. Both consist of vertical bars (called **bins**) with heights that represent the number of data points (called the **frequency**) in each bin. In a **histogram** each bar represents the number of data elements within a certain range of values. Values at the left side of a bin's range are included in that bin. Each range of values should have the same width. In a **bar graph** each bar represents the number of data elements in a certain category. All the bars are the same width and are separated from each other. For additional information and examples, see the Math Notes box on page 8 in the text.

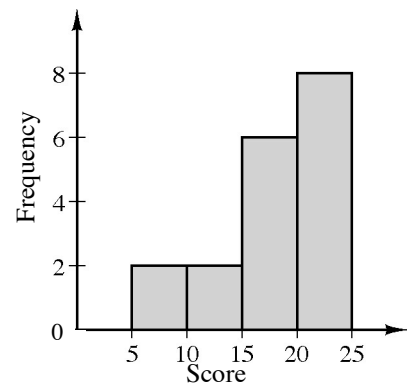
Example 1

The scores for a 25-point quiz are listed below arranged from least to greatest.

7, 7, 12, 13, 15, 16, 16, 16, 18, 19, 20, 20, 20, 21, 21, 22, 23, 23, 24

Using intervals of five points, create a histogram for the class.

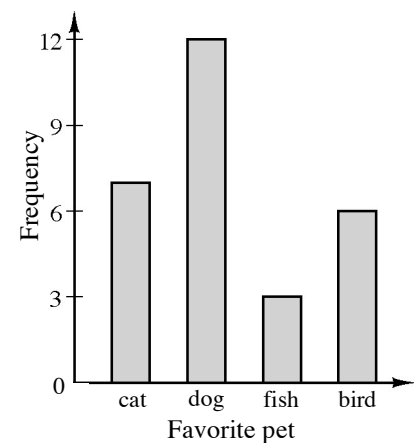
See histogram at right. Scores on the right end of the interval are included in the next interval. The interval between 10 and 15 only includes the two scores of 12 and 13. The interval between 15 and 20 only includes the six scores of 15, 16, 16, 16, 18, and 19.



Example 2

Ms. Lim asked each of her students about their favorite kind of pet. Based on their responses, she drew the bar graph at right. Use the bar graph to answer each question.

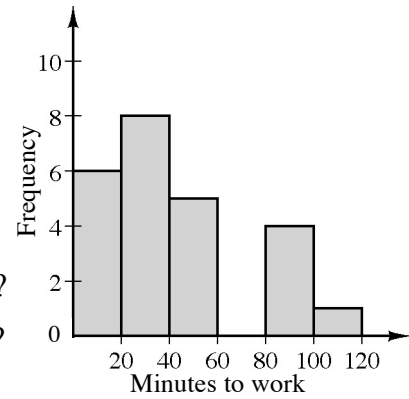
- What is the favorite pet?
- How many students chose a bird as their favorite pet?
- What was the least favorite pet?
- If every student voted once, how many students are in the class?



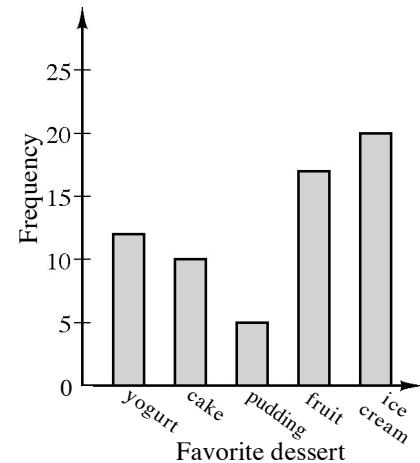
Answers: a. dog b. 6 c. fish d. 28

Problems

- Mr. Diaz surveyed his employees on the time it takes them to get to work. The results are shown in the histogram at right.
 - How many employees completed the survey?
 - How many employees get to work in less than 20 minutes?
 - How many employees get to work in less than 40 minutes?
 - How many employees take 60 minutes to get to work?



- The two sixth grade classes at Vista Middle School voted for their favorite dessert. The results are shown in the bar graph at right for the five favorite choices.
 - What was the favorite dessert and how many students made that choice?
 - How many students selected cake as their favorite dessert?
 - How many students selected yogurt as their favorite?
 - How many more students selected ice cream than pudding?



- Mr. Fernandez asked 30 people at work how many pets they owned. The results are shown below. Make a histogram to display this data. Use intervals of one pet.

0 pets	5 people
1 pet	8 people
2 pets	10 people
3 pets	3 people
4 pets	2 people
5 pets	1 person
9 pets	1 person

- During the first week of school Ms. Chan asked her students to name the county where they were born. There were so many different countries she grouped them by continent:

North America: 14 students, South America: 2 students, Europe: 3 students,
Asia: 10 students, Africa: 1 student, Australia: 0 students.

Make a bar graph to display this information.

5. Three coins were tossed 20 times and the number of results that were “heads” each time is shown below:

1, 1, 2, 0, 2, 3, 1, 2, 1, 2, 2, 1, 3, 2, 0, 1, 2, 0, 2, 1

Make a histogram to show the results.

6. The physical education teacher at West Middle School asked the class about their favorite winter activity. Here were the results:

reading: 8 students, ice skating: 4 students, skiing: 6 student,
snowboarding: 11 students, computer activities: 14 students.

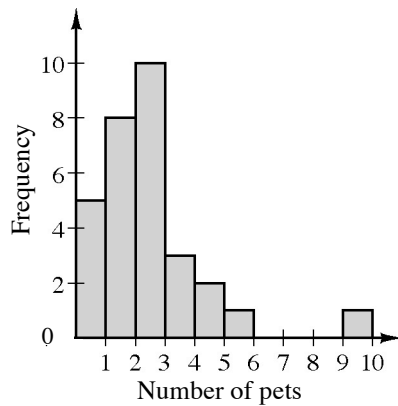
Make a bar graph to show the results.

Answers

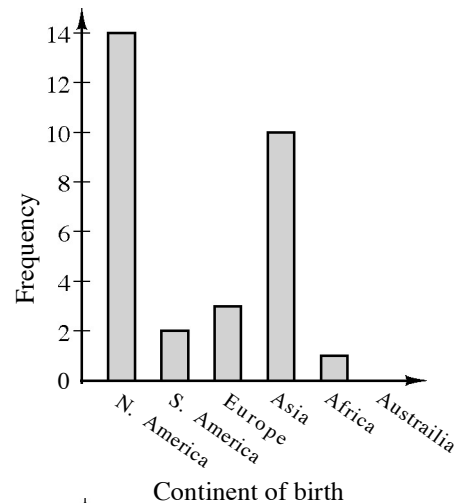
1. a. 24 b. 6 c. 14 d. 0

2. a. ice cream 20 b. 10
c. 12 d. 15

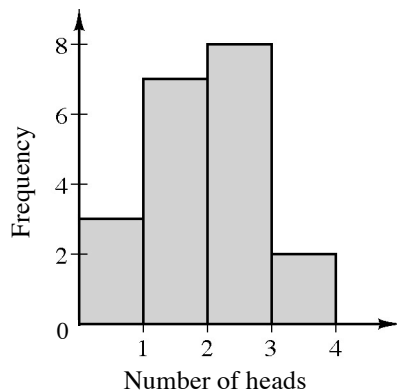
3.



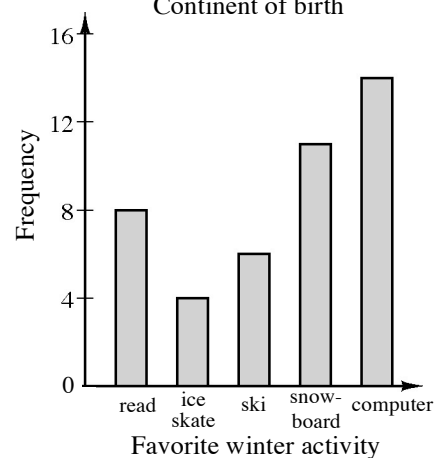
4.



5.



6.

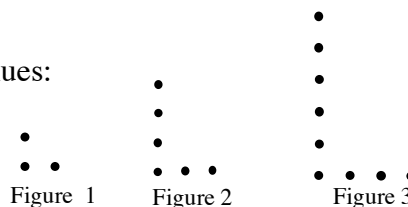


Students are asked to use their powers of observation and pattern recognition skills to extend patterns and predict the number of dots that will be in a figure that is too large to draw. Later in the course, variables will be used to describe the patterns.

Example

Examine the dot pattern at right. Assuming the pattern continues:

- a. Draw figure 4.
- b. How many dots will be in Figure 10?



Solution:

The horizontal dots are one more than the figure number and the vertical dots are even numbers (or, twice the figure number).

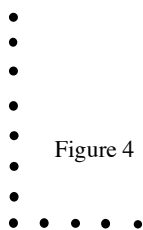


Figure 1 has 3 dots, Figure 2 has 6 dots, and Figure 3 has 9 dots. The number of dots is the figure number multiplied by three.

Figure 10 has 30 dots.

Problems

For each dot pattern, draw the next figure and determine the number of dots in Figure 10.

1. Figure 1 Figure 2 Figure 3

2. Figure 1 Figure 2 Figure 3 Figure 4

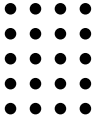
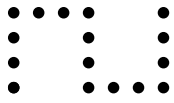
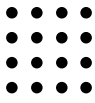

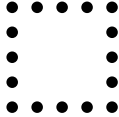
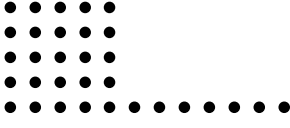
3. Figure 1 Figure 2 Figure 3

4. Figure 1 Figure 2 Figure 3

5. Figure 1 Figure 2 Figure 3

6. Figure 1 Figure 2 Figure 3

Answers

1.  50 dots 2.  31 dots 3.  110 dots
4.  22 dots 5.  40 dots 6.  142 dots
- Figure 4 Figure 5 Figure 4 Figure 4 Figure 5 Figure 4

PROPORTIONAL RELATIONSHIPS

1.1.5

Students solve proportional reasoning (ratio) problems in a variety of ways. They may find the number or cost for one unit and they multiply by the number of units. They may also organize work in a table. Later in the course students will use ratio equations or proportions to solve this kind of problem.

Example 1

If three boxes of cereal weigh 36 ounces, how much will 10 boxes weigh?

$$36 \div 3 = 12 \text{ ounces per box}$$

$$12 \cdot 10 = 120 \text{ ounces}$$

Example 2

If 10 pencils weigh 34 grams, how much will 50 pencils weigh?

$$x 5 \left\{ \begin{array}{c|c} \hline \# \text{ pencils} & \text{ounces} \\ \hline 10 & 34 \\ \hline 50 & ? \\ \hline \end{array} \right\} x 5$$

$$34 \cdot 5 = 170 \text{ ounces}$$

Problems

Solve each problem and explain your reasoning.

1. Alice knows six cups of rice will make enough Spanish rice to feed 15 people. How much rice is needed to feed 75 people?
2. Elaine can plant 16 flowers in 10 minutes. How many can she plant in 30 minutes?
3. Ivanna needs to buy 36 cherries for a salad. She can buy nine cherries for \$0.54. How much will 36 cherries cost?
4. A plane travels 3400 miles in eight hours. How far would it travel in six hours at the same rate?
5. Leslie can write a 450-word essay in 1 hour. If she writes an essay in 20 minutes, approximately how many words should the essay contain?
6. About eight out of every 100 people in the state have red hair. If a typical classroom in the state has 25 students, how many would you expect to have red hair?
7. When Carlos rides his bike to school, it takes 15 minutes to go 8 blocks. If he rides at the same speed, how long should it take him to travel 30 blocks?
8. Simba the cat is on a diet. Ten pounds of special low-fat food costs \$22.50. How much would 30 pounds cost? How much would 36 pounds cost?
9. Elizabeth came to bat 110 times in 20 games. How many times should she expect to bat in 70 games?
10. Ly can deliver 32 newspapers in 25 minutes on his bike. Next week he needs to deliver 80 newspapers in the same neighborhood. How long should it take him if he works at the same rate as he did for 32 newspapers?

Answers

- | | | | | |
|-------------|---------------|---------------------|----------------|---------------|
| 1. 30 cups | 2. 48 flowers | 3. \$2.16 | 4. 2550 miles | 5. 150 words |
| 6. 2 people | 7. 56.25 min. | 8. \$67.50; \$81.00 | 9. 385 at-bats | 10. 62.5 min. |

When counting large numbers of one item or comparing two quantities of the same item, it is helpful to have the items arranged in a way that is easy to count. In comparing two quantities, they may be described as equal ($=$), or one is greater than ($>$), or less than ($<$) the other.

Example 1

Write an expression that represents the total number of items and give the total.

$4(50) + 3(5) + 4 = 200 + 15 + 4 = 219$

Example 2

Write expressions to present the total number of items in each pile and then compare the totals using $=$, $<$, or $>$.

<p>Pile #1</p> <p> $6(15) + 9 + 3$ $92 < 96$ </p>	<p>Pile #2</p> <p> $16(6)$ </p>
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Problems

Write an expression that represents the total number of items and give the total.

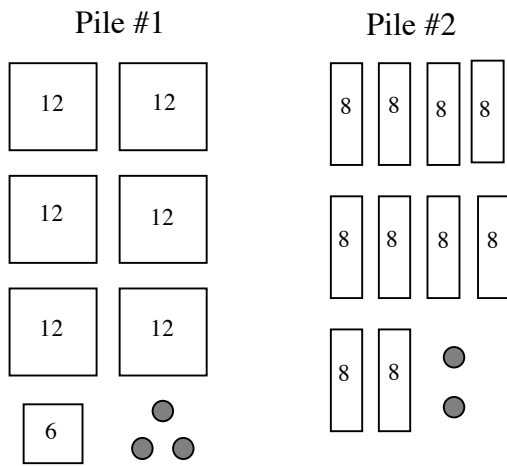
1.

2.

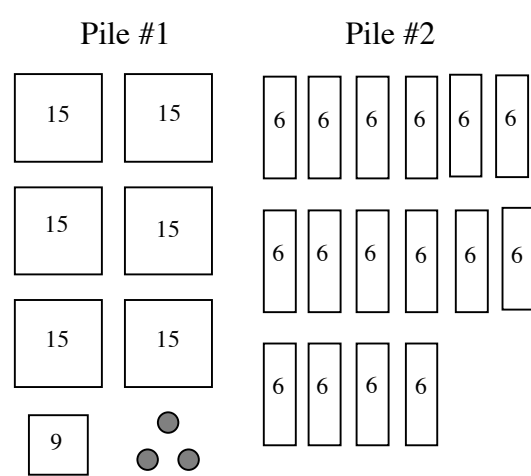
3.

Write expressions to present the total number of items in each pile and then compare the totals using =, <, or >.

4.



5.



Answers

1. $4(40) + 2(4) + 2 = 170$

2. $3(20) + 4(80) + 2(5) = 390$

3. $2(60) + 6 + 5 = 131$

4. $81 < 82$

5. $102 > 96$

When two or more integers are multiplied together, each number is a **factor** of the product. Non-negative integers that have exactly two factors, namely, one and itself, are called **prime** numbers. Except for one and zero, the other non-prime numbers are **composite**. One has only one factor, so it is neither prime nor composite. It is also the **multiplicative identity** since one multiplied by any number does not change the value. Written in symbols, $1 \cdot n = n$. Zero is the additive identity since adding zero to any number does not change the value. Written in symbols, $0 + n = n$. For more information, see the Math Notes box on page 38 of the text.

Properties of numbers are also discussed in sections 2.2.1, 2.2.2, and 3.4.5 of the text.

Example 1

Identify each integer as prime, composite, or neither.

- 6 6 has factors of 1, 6, 2, and 3 so 6 is composite.
- 17 17 has factors of 1 and 17 so 17 is prime.
- 1 1 has only 1 as a factor so 1 is neither.

Example 2

For each composite number, factor it into primes and write the number as a product of primes using exponents as possible.

- 24 $24 = 4 \cdot 6 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$
- 45 $45 = 9 \cdot 5 = 3 \cdot 3 \cdot 5 = 3^2 \cdot 5$

Problems

Identify each integer as prime, composite, or neither. For each composite number, factor it into primes and write the number as a product of primes, using exponents as possible.

- | | | | |
|-------|-------|--------|--------|
| 1. 30 | 2. 15 | 3. 16 | 4. 20 |
| 5. 11 | 6. 38 | 7. 29 | 8. 100 |
| 9. 53 | 10. 0 | 11. 54 | 12. 96 |

Answers

- | | | | |
|------------------------|-----------------|-------------------|--------------------|
| 1. $2 \cdot 3 \cdot 5$ | 2. $3 \cdot 5$ | 3. 2^4 | 4. $2^2 \cdot 5$ |
| 5. prime | 6. $2 \cdot 19$ | 7. prime | 8. $2^2 \cdot 5^2$ |
| 9. prime | 10. neither | 11. $2 \cdot 3^3$ | 12. $2^5 \cdot 3$ |

A **stem-and-leaf plot** is a way to display data that shows each individual value from a set of numbers and how they are distributed. The vertical “stem” part of the graph represents all of the digits in a number except the last one and the horizontal “leaves” represent the last digit of the number. For an additional example, see the Math Notes box on page 48 in the text.

Example 1

Make a stem-and-leaf plot of this set of data:
34, 31, 37, 44, 38, 29, 34, 42, 43, 34, 52, and 41.

Solution:

$$\begin{array}{r|l} 2 & 9 \\ 3 & 1\ 4\ 4\ 4\ 7\ 8 \\ 4 & 1\ 2\ 3\ 4 \\ 5 & 2 \end{array}$$
Example 2

Make a stem-and-leaf plot of this set of data: 192, 182, 180, 192, 178, 175, 195, 177, and 177.

Solution:

$$\begin{array}{r|l} 17 & 5\ 7\ 7\ 8 \\ 18 & 0\ 2 \\ 19 & 2\ 2\ 5 \end{array}$$
Problems

Make a stem-and-leaf plot of each set of data.

1. 29, 28, 34, 30, 33, 26, 18, and 34.
2. 2.5, 3.4, 2.7, 2.5, 1.9, 3.1, 4.2, and 3.0.
3. 80, 89, 79, 84, 95, 79, 89, 67, 82, 76, 92, 89, 81, and 123.
4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.
5. 45, 47, 52, 85, 46, 32, 83, 80, and 75.
6. 475, 462, 456, 480, 472, 455, 454, and 480.
7. 49, 54, 52, 58, 61, 72, 73, 78, 73, 82, 83, 73, 61, 67, and 68.
8. 65, 35, 48, 29, 57, 87, 94, 68, 86, 73, 58, 74, 85, 91, 88, and 97.

Answers

1.

$$\begin{array}{r|l} 1 & 8 \\ 2 & 6\ 8\ 9 \\ 3 & 0\ 3\ 4\ 4 \end{array}$$

2.

$$\begin{array}{r|l} 1 & 9 \\ 2 & 5\ 5\ 7 \\ 3 & 0\ 1\ 4\ 5 \\ 4 & 2 \end{array}$$

3.

$$\begin{array}{r|l} 6 & 7 \\ 7 & 6\ 9\ 9 \\ 8 & 0\ 1\ 2\ 4\ 9\ 9\ 9 \\ 9 & 2\ 5 \\ 10 & \\ 11 & \\ 12 & 3 \end{array}$$

4.

$$\begin{array}{r|l} 9 & 1 \\ 10 & 0\ 1\ 1\ 3\ 4\ 5\ 7\ 8\ 9 \\ 11 & 1\ 3\ 4\ 6\ 8 \end{array}$$

5.

$$\begin{array}{r|l} 3 & 2 \\ 4 & 5\ 6\ 7 \\ 5 & 2 \\ 7 & 5 \\ 8 & 0\ 3\ 5 \end{array}$$

6.

$$\begin{array}{r|l} 45 & 4\ 5\ 6 \\ 46 & 2 \\ 47 & 2\ 5 \\ 48 & 0\ 0 \end{array}$$

7.

$$\begin{array}{r|l} 4 & 9 \\ 5 & 2\ 4\ 8 \\ 6 & 1\ 1\ 7\ 8 \\ 7 & 2\ 3\ 3\ 3\ 8 \\ 8 & 2\ 3 \end{array}$$

8.

$$\begin{array}{r|l} 2 & 9 \\ 3 & 5 \\ 4 & 8 \\ 5 & 7\ 8 \\ 6 & 5\ 8 \\ 7 & 3\ 4 \\ 8 & 5\ 6\ 7\ 8 \\ 9 & 1\ 4\ 7 \end{array}$$

The **measures of central tendency** are numbers that locate or approximate the “center” of a set of data. Mean, median and mode are the most common measures of central tendency.

The **mean** is the arithmetic average of a data set. Add all the values in a set and divide this sum by the number of values in the set. The **median** is the middle number in a set of data arranged numerically. The **mode** is the value in the data set that occurs more often than any other value. An **outlier** is a number that is much smaller or larger than most of the others in the data set. The **range** of a data set is the difference between the highest and lowest values of the data set.

See the Math Notes box on page 56 of the text for more information and an additional example.

The **mean** is calculated by finding the sum of the data set and dividing it by the number of elements in the set.

Example 1

Find the mean of this set of data: 34, 31, 37, 44, 38, 34, 42, 34, 43, and 41.

- $34 + 31 + 37 + 44 + 38 + 34 + 42 + 34 + 43 + 41 = 378$
- $378 \div 10 = 37.8$

The mean of this set of data is 37.8.

Example 2

Find the mean of this set of data: 92, 82, 80, 92, 78, 75, 95, and 77.

- $92 + 82 + 80 + 92 + 78 + 75 + 95 + 77 = 748$
- $748 \div 9 = 83.1$

The mean of this set of data is 83.1.

Problems

Find the mean of each set of data.

1. 29, 28, 34, 30, 33, 26, and 34.
2. 25, 34, 35, 27, 31, and 30.
3. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.
4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The **mode** is the value in a data set that occurs most often. Data sets may have more than one mode or no mode.

Example 3

Find the mode of this set of data: 34, 31, 37, 44, 34, 42, 34, 43, and 41.

- The mode of this data set is 34 since there are three 34's and only one of each of the other numbers.

Example 4

Find the mode of this set of data: 92, 82, 80, 92, 78, 75, 95, 77, and 77.

- The modes of this set of data are 77 and 92 since there are two of each of these numbers and only one of each of the other numbers. This data set is said to be **bimodal** since it has two modes.

Problems

Find the mode of each set of data.

5. 29, 28, 34, 30, 33, 26, and 34.
6. 25, 34, 35, 27, 25, 31, and 30.
7. 80, 89, 79, 84, 95, 79, 89, 76, 82, 76, 92, 89, 81, and 123.
8. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The **median** is the middle number in a set of data arranged in numerical order. If there are an even number of values, the median is the mean (average) of the two middle numbers.

Example 5

Find the median of this set of data: 34, 31, 37, 44, 38, 34, 43, and 41.

- Arrange the data in order: 31, 34, 34, 34, 37, 38, 41, 43, and 44.
- Find the middle value(s): 37 and 38.
- Since there are two middle values, find their mean: $37 + 38 = 75$, $75 \div 2 = 37.5$. Therefore, the median of this data set is 37.5.

Example 6

Find the median of this set of data: 92, 82, 80, 92, 78, 75, 95, 77, and 77.

- Arrange the data in order: 75, 77, 77, 78, 80, 82, 92, 92, and 95.
- Find the middle value(s): 80. Therefore, the median of this data set is 80.

Problems

Find median of each set of data.

9. 29, 28, 34, 30, 33, 26, and 34.
10. 25, 34, 27, 25, 31, and 30.
11. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.
12. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The **range** of a set of data is the difference between the highest value and the lowest value.

Example 7

Find the range of this set of data: 114, 109, 131, 96, 140, and 128.

- The highest value is 140.
- The lowest value is 96.
- $140 - 96 = 44$.
- The range of this set of data is 44.

Example 8

Find the range of this set of data: 37, 44, 36, 29, 78, 15, 57, 54, 63, 27, and 48.

- The highest value is 78.
- The lowest value is 27.
- $78 - 27 = 51$.
- The range of this set of data is 51.

Problems

Find the range of each set of data in problems 9 through 12.

Outliers are numbers in a data set that are either much higher or much lower than the other numbers in the set.

Example 9

Find the outlier of this set of data: 88, 90, 96, 93, 87, 12, 85, and 94.

- The outlier is 12.

Example 10

Find the outlier of this set of data: 67, 54, 49, 76, 64, 59, 60, 72, 123, 44, and 66.

- The outlier is 123.

Problems

Find the outlier for each set of data.

13. 70, 77, 75, 68, 98, 70, 72, and 71.

14. 14, 22, 17, 61, 20, 16, and 15.

15. 1376, 1645, 1783, 1455, 3754, 1790,
1384, 1643, 1492, and 1776.

16. 62, 65, 93, 51, 55, 14, 79, 85, 55, 72, 78,
83, 91, and 76.

Answers

1. 30.57

2. $30.\bar{3}$

3. $86.\bar{13}$

4. $106.\bar{6}$

5. 34

6. 25

7. 89

8. 101 and 113

9. median 30;
range 8

10. median 28.5;
range 9

11. median 82;
range 47

12. median 107;
range 27

13. 98

14. 61

15. 3754

16. 14