Sequences & Series Practice Test Name________________

Write out the first five terms of the sequence.

1) $a_n = n^2 - n$

$0, 2, 6, 12, 20$

2) $c_n = \frac{n+2}{n}$

$3, 2, 5/3, 3/2, 7/5$

Find the first six terms of the sequence.

3) $a_1 = -2, a_n = 3 \cdot a_{n-1}$

$-2, -6, -18, -54, -162, -486$

4) $a_1 = 3, a_2 = 3$; for $n \geq 3, a_n = a_{n-1} + a_{n-2}$ → a_n = prev term + "prev-prev" term

$3, 3, 6, 9, 15, 24$

Find an explicit rule for the nth term of the sequence.

5) 7, −7, 7, −7, ... $r = -1$ $g_1 = 7$

$g_n = 7 \cdot (-1)^{(n-1)}$

Find an explicit rule for the nth term of the arithmetic sequence.

6) −10, −20, −30, −40, ... $d = -10$ $a_1 = -10$

$a_n = -10 - 10(n-1)$ OR... $a_n = -10n$

Find an explicit rule for the nth term of the sequence.

7) The second and fifth terms of a geometric sequence are −24 and 1536, respectively.

$g_n = 6 \cdot (-4)^{(n-1)}$

$6, -24, \_, \_, 1536$

$1536 = -24 \cdot r^3$ $-64 = r^3$ $r = -4$

Solve.

8) A certain radioactive isotope has a half-life of 15 days. If one is to make a table showing the half-life decay of a sample of this isotope from 32 grams to 1 gram; list the time (in days, starting with t=0) in the first column and the mass remaining (in grams) in the second column, which type of sequence is used in the first column and which type of sequence is used in the second column?

A) Geometric in the first; geometric in the second
B) Arithmetic in the first; arithmetic in the second
C) Geometric in the first; arithmetic in the second
D) Arithmetic in the first; geometric in the second

9) A certain species of tree grows an average of 4.2 cm per week. Write an explicit rule for the sequence that represents the weekly height of this tree in centimeters if the measurements begin when the tree is 3 meters tall.

3m = 300cm $a_n = 300 + 4.2(n)$

Begins = a_0

Find a recursive rule for the nth term of the sequence.

10) −8, 3, 14, 25, ...

$$\begin{cases} a_1 = -8 \\ a_n = a_{n-1} + 11 \end{cases}$$

Determine whether the sequence converges or diverges. If it converges, give the limit.

11) $\frac{1}{462}, \frac{1}{42}, \frac{11}{42}, \frac{121}{42}, \ldots$ $r = 11$ diverges

12) 4, 12, 36, 108, ... $r = 3$ diverges

Determine whether the infinite geometric series converges. If the series converges, determine the limit.

13) 72 − 36 + 18 − 9 + ... $r = (-1/2)$ converges to 48

14) $\frac{1}{262{,}144} + \frac{1}{32{,}768} + \frac{1}{4096} + \frac{1}{512} + \ldots$

$r = 8$ diverges

Write the sum using summation notation, assuming the suggested pattern continues.

15) 2 − 8 + 32 − 128 + ...

$$\sum_{n=1}^{\infty} 2 \cdot (-4)^{n-1}$$

16) 17 + 19 + 21 + 23 + ... + (2n+1) ...

$$\sum_{n=8}^{\infty} 2n+1$$

$2n+1 = 17$

$2n = 16 \quad n = 8$

Find the sum of the arithmetic sequence.

17) 251, 249, 247, 245, ..., 233

$$\sum_{n=1}^{10} 253 - 2n$$

$\frac{10}{2}(2 \cdot 251 - 2(10-1))$

$\boxed{2420}$

$251 - 2(n-1) = 253 - 2n$

$233 = 253 - 2n$

$-20 = -2n$ so $n = 10$

18) 56, 58, 60, 62, ..., 78

$$\sum_{n=1}^{12} 54 + 2n$$

$\frac{12}{2}(2 \cdot 56 + 2(12-1))$

$\boxed{804}$

$56 + 2(n-1) = 54 + 2n$

$54 + 2n = 78 \quad n = 12$

Find the sum of the geometric sequence.

19) 2, −8, 32, −128, 512

$\boxed{410}$

20) $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{16}{3}$

$\boxed{\frac{31}{3}}$

Solve.

21) The population of a town was 25,600 at the beginning of 1970. If the population decreased 250 people per year, how many people lived in the town at the beginning of 1985?

let $a_1 = 1970$'s pop

$n = 16$

$25{,}600 - 250(n-1)$

$25600 - 250(16-1)$ 21,850 people

22) A ball is dropped from a height of 5.0 m. On each upward bounce the ball returns to $\frac{4}{5}$ of its previous height. Find the total vertical distance the ball travels before coming to rest.

↑ and ↓

$\lim_{n \to \infty} 4\left(\frac{4}{5}\right)^{(n-1)}$

$\frac{4}{1-\frac{4}{5}} = 20 \times 2$ (up & down)

$40 + 5$ (drop ht)

$\boxed{45 \text{ m total}}$

23) A pendulum bob swings 4.0 cm on its first oscillation. On each subsequent oscillation the bob travels $\frac{1}{2}$ of the previous distance. Find the total distance the bob travels before coming to rest.

Kind of like drop height

$\sum_{n=1}^{\infty} 2.0\left(\frac{1}{2}\right)^{n-1} = \frac{2}{\left(1-\frac{1}{2}\right)} = 4 \times 2 = 8$ (swings → ←)

$+ 4$ 1st swing

$\boxed{12 \text{ cm}}$

go → 4, 2, 1, 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128, ...