3.1.1/3.1.2

Exponential Functions and Logistic Functions

Precalculus 12-13

ESSENTIAL UNDERSTANDINGS

- Exponential functions model **unrestricted** growth/decay over time
- Logistic functions model **restricted** growth/decay over time
- Logarithmic functions model things like the Richter scale and the decibel scale

GENERAL DEFINITION - EXPONENTIAL

- $f(x) = a \cdot b^x$
  - $a = initial\ condition$
  - $b = base, growth\ decay\ factor$
    - If $b > 1$, it models exponential growth
    - If $0 < b < 1$, it models exponential decay
    - $b = 1 + r$, where *r* represents the constant percentage rate
  - $x = independent\ variable$, usually representing time

IS THE FOLLOWING AN EXAMPLE OF AN EXPONENTIAL FUNCTION?

- $g(x) = x^8$
- $d(x) = 3^x$
- $f(x) = 4^4$

EXPONENTIAL POPULATION MODEL

- If P is changing at a constant percentage rate *r* each year, then the data can be described by

$$\mathrm{P}(t) = P_0(1 + r)^t$$

IDENTIFY THE FOLLOWING

- $f(x) = 78{,}963(0.986)^x$
- $g(t) = 43(2.5)^t$

DETERMINE THE EXPONENTIAL MODEL

- Initial value: 5
- Decreasing at a rate of 0.59% per week

x	*y*
-2	1.472
-1	1.84
0	2.3
1	2.875
2	3.59375

ANALYZE THE FUNCTION

- $w(x) = 4(0.5)^x$

- The half-life of a certain radioactive substance is 65 days. There are 3.5 g present initially. Write the formula to model this situation.

- When will there be less than 1 g remaining?

THE NATURAL BASE

- $e \approx 2.618281828459$
- Named after Leonhard Euler, who introduced the notation
- $f(x) = e^x$ has special calculus properties that simply many calculations, thus e is called the natural base
- Used most frequently in compounding continuous formulas, such as $A = Pe^{rt}$

- The number of bacteria B found in a petri dish is given by the formula

$$B = 100 \cdot e^{0.693t}$$

What is the number of bacteria initially present?

How many bacteria are present after 5.5 hours?

- The amount *C* of carbon-14 (in grams) after *t* years is given by $C = 20e^{-0.0001216t}$. Estimate the half-life of carbon-14.

3.1.2

LOGISTIC FUNCTIONS

LOGISTIC FUNCTIONS

- In many growth situations, there is a limit to possible growth. The growth begins as an exponential manner, but eventually slows and the graph levels out, causing horizontal asymptotes
- Graph is bounded (see pg. 89 for definition)

General Formula

- $h(x) = \frac{c}{1+a \cdot b^x}$
- c is the limit to growth (a constant)

LOGISTIC FUNCTIONS ARE BOUNDED

- $\lim_{x\to-\infty} f(x) = 0$
- $\lim_{x\to\infty} f(x) = c$; maximum sustainable population
- Has a range of (0, c)
- Two horizontal asymptotes $y = 0, y = c$, given the function has no vertical translation
- See pg. 259 for more analysis

ANALYZE

- $f(t) = \frac{12}{1+2\cdot 0.8^{x}}$

FIND THE LOGISTIC EQUATION

- Initial population: 16, maximum sustainable population: 128, passing through (5, 32).

- The population of deer after t years is modeled by the function $D(t) = \frac{1001}{1+90 \cdot e^{-0.2t}}$.

What is the initial population of deer?

When will the deer population be 700?

What is the maximum sustainable population?