Arithmetic sequences

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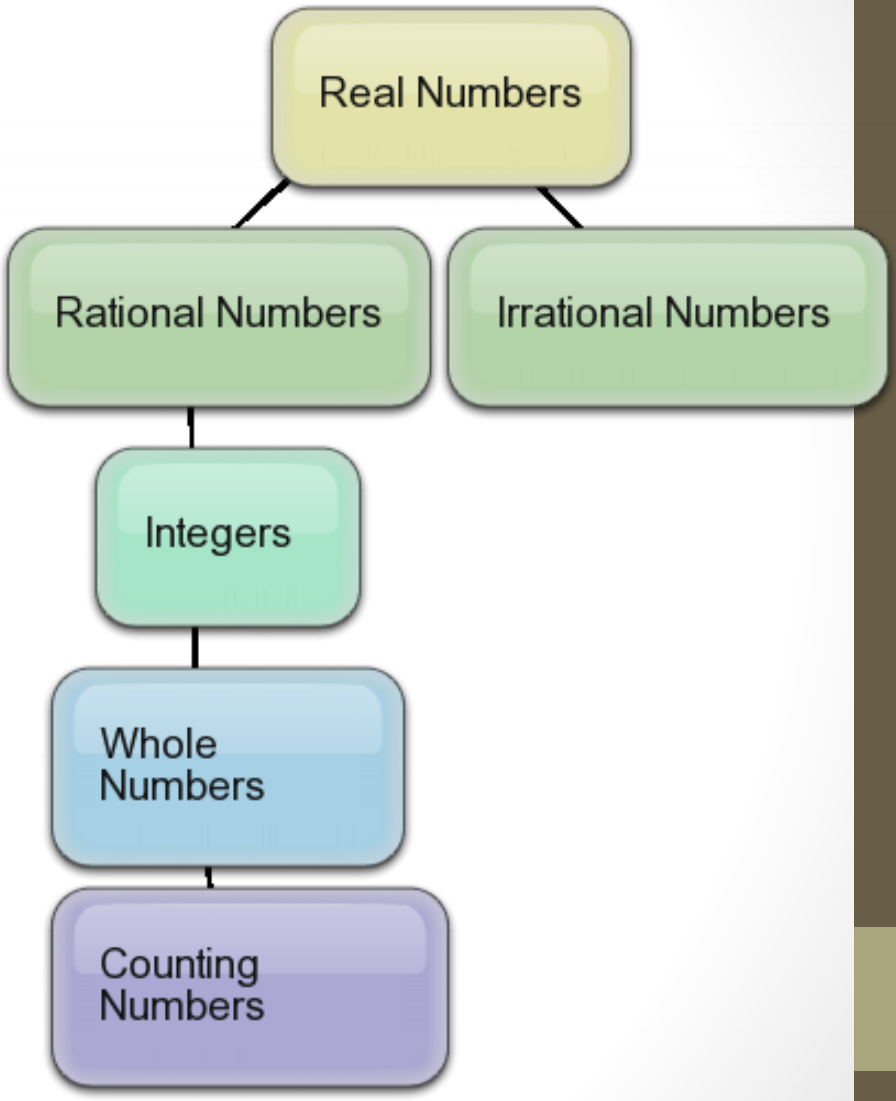
What is a sequence?

- A list of values having a specific pattern
  - 10, 20, 30, 40…
  - 5, 10, 20, 40, 80, …
- The domain of a sequence is the set of natural numbers

What is a domain?

- *Domain* is the allowable inputs for a variable, such as *x*.
- The "natural" numbers are also called the counting numbers
- 1, 2, 3, 4, …



- 1, 4, 9, 16, …
- 3, 6, 9, 12, …
- 7, 9, 11, 13, …
- 100, 50, 25, 12.5, …
- 2, 4, 8, 16, …

Describe the pattern

Sequence vocabulary

- ***Term:*** a value found in a sequence
- ***Index:*** the position of a term in a sequence (the ***ordinal*** numbers – first, second, third, etc)

Sequence notation

- n = the term position, the index
- a_n = the "n^{th}" term
  - Used to describe a non-specific term in a sequence
    - a_2 = the second term in sequence "a"
    - h_{12} = the twelfth term in sequence "h"

Two types of sequences

Arithmetic:

- Pattern is either addition or subtraction from one term to the next
- Constant difference "*d*"

Geometric:

- Pattern is either multiplication or division
- Constant ratio "*r*"

Two types of formulas

EXPLICIT

- Is a “standard equation” like slope-intercept form or exponential form
- Allows you to find the value of any term in a sequence
  - Substitute a number for “n”

RECURSIVE

- Has two parts:
  - It must tell the reader the first term of the sequence
  - It must tell the reader the pattern to get from one term to the next

You should be able to translate between these two formulas for any given sequence.

Formulas for an arithmetic sequence

Explicit

- $a_n = a_1 + d(n - 1)$, $where \quad n \geq 2$
- d= constant difference
- n stays the variable, represents the index
- This can be simplified using the Distributive Property

Recursive

- $\begin{cases} first\ term\ is\ defined \\ pattern\ is\ described \end{cases}$
- Usually looks like this:

$$\begin{cases} s_1 = -5 \\ s_n = (s_{n-1}) + 11 \end{cases}$$

- s_{n-1} literally means “the previous term” or “the term before”

Write the explicit and recursive formulas for each sequence

98, 92, 86, 80, …	3, 6, 9, 12, …	7, 9, 11, 13, …

A hot air balloon is 10’ feet off the ground when the ropes are released. The balloon rises at a rate of 3 feet per second.

Describe this situation recursively	What is the height of the balloon after the sixth second?	When is the balloon 73 feet high?

You've opened a bank account with $1,400 and plan to withdraw $35 each week.

- Describe the situation explicitly

- When will you not be able to withdraw any more money?

Geometric sequences

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Sequences in a table

n	1	2	3	4	5…
	a_1	a_2	a_3	a_4	a_5
a_n	6	16	26	36	46…

Explicit Formula:

Find the first six terms

$$\begin{cases} a_1 = 19 \\ a_n = a_{n-1} + 4 \end{cases}$$

Formulas for a Geometric sequence

Explicit

- $g_n = g_1 * r^{(n-1)}$
- r= constant ratio
- n stays the variable, represents the index
- This ***cannot*** be simplified using the Distributive Property

Recursive

- $\begin{cases} first\ term\ is\ defined \\ pattern\ is\ described \end{cases}$

Write the recursive & explicit formulas

100, 50, 25, 12.5, …

2, 4, 8, 16, …

Bouncing Ball problem

A ball is dropped from a height of 5 feet (this is NOT a bounce). Each bounce reaches 75% of its previous height.

- Draw a picture of this situation
- Write an explicit formula to model this situation.
- Use this formula to determine the height of the tenth bounce.

Hot Hot-tub

You complain that the hot tub in your hotel suite is not hot enough. The concierge tells you she will increase the temperature by 10% each hour. If the current temperature of the hot tub is 75° F, what will be the temperature of the hot tub after 3 hours, to the *nearest tenth* of a degree?

Arithmetic or geometric?

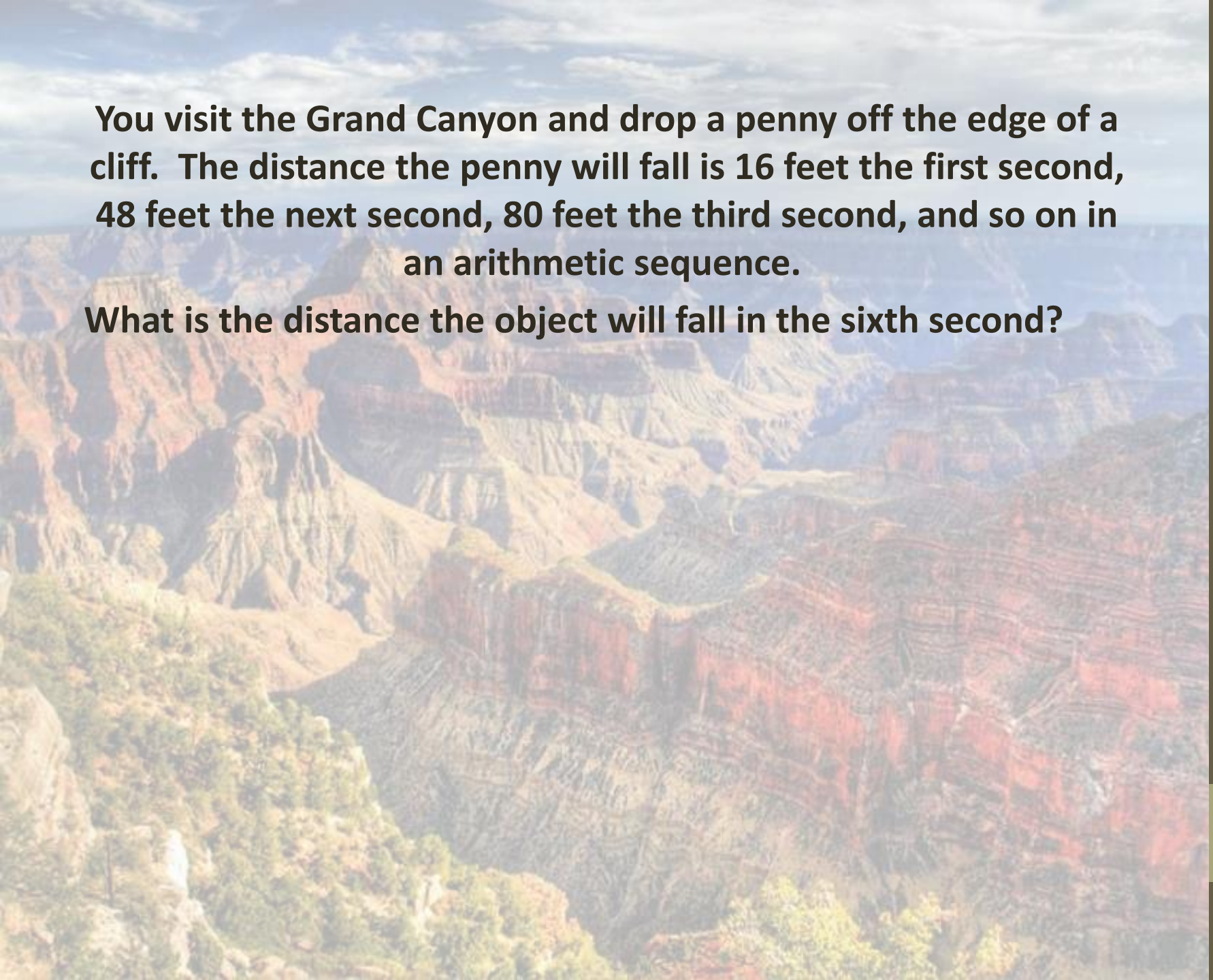
A mine worker discovers an ore sample containing 500 mg of radioactive material. It is discovered that the radioactive material has a half life of 1 day.

Find the amount of radioactive material in the sample at the beginning of the 7^{th} day.

Arithmetic or geometric?

After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day.

How many weeks will it be before you are up to jogging 60 minutes per day?



You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on in an arithmetic sequence.

What is the distance the object will fall in the sixth second?