

The six trigonometric functions/Reciprocals Identities

$$\sin x = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} = \frac{1}{\csc x}$$

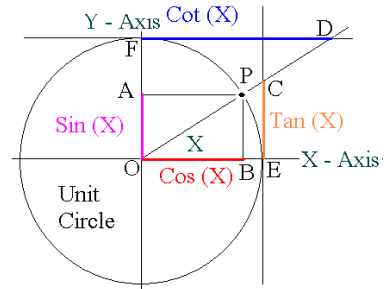
$$\cos x = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} = \frac{1}{\sec x}$$

$$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} = \frac{\sin x}{\cos x}$$

$$\csc x = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{1}{\sin x}$$

$$\sec x = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = \frac{1}{\cos x}$$

$$\cot x = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + \sec^2 x = -1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Sum or difference of two angles

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Double Angle Formulas

$$\cos 2x = 1 - 2\sin^2 x \quad \cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = \cos^2 x - \sin^2 x \quad \sin 2x = 2\sin x \cos x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half Angle Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2} \quad \tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Reduction Formulas

$$\sin(-x) = -\sin x$$

$$\sin(x) = -\sin(x - \pi)$$

$$\tan(-x) = -\tan x$$

$$\pm \sin x = \cos\left(x \pm \frac{\pi}{2}\right)$$

$$\cos(-x) = \cos x$$

$$\cos(x) = -\cos(x - \pi)$$

$$\tan(x) = \tan(x - \pi)$$

$$\pm \cos x = \sin\left(x \pm \frac{\pi}{2}\right)$$

Law of cosines $a^2 = b^2 + c^2 - 2bc \cos A$

(where A is the angle of a scalene Δ opposite side a)

Co-Functions

$$\sin x = \cos(90^\circ - x)$$

$$\cos x = \sin(90^\circ - x)$$

$$\tan x = \cot(90^\circ - x)$$

$$\sec x = \csc(90^\circ - x)$$

Sum and Product Formulas

$$\sin a \cos b = \frac{1}{2}[\sin(a + b) + \sin(a - b)]$$

$$\cos a \sin b = \frac{1}{2}[\sin(a + b) - \sin(a - b)]$$

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos a \cos b = \frac{1}{2}[\cos(a + b) + \cos(a - b)]$$

$$\sin a \sin b = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$